Least-Squares Carrier Frequency Offset Estimation for Coherent Optical QPSK Receivers

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Abstract—**We investigate a carrier frequency offset (CFO) estimation algorithm for coherent optical quaternary phase-shift keying (QPSK) receivers. The algorithm utilizes the phase difference between the samples with different time-domain separations, so that the estimation accuracy can be significantly enhanced, compared with the algorithm that only compares the phase difference between adjacent samples. In a 28-GBaud coherent QPSK system, simulation results show that when 16 time spans and 512 samples are utilized, a uniform standard deviation of less than 3 MHz can be achieved for the CFO estimation error, when the optical signal-to-noise ratio is 11.5 dB.**

Keywords—carrier frequency offset; coherent optical fiber communication; least-squares method; quaternary phase-shift keying

I. INTRODUCTION

Coherent detection in optical fiber communication systems is a powerful technique that has recently attracted much attention in both academic and industrial fields. It not only enables complex modulation formats that utilize both the amplitude and the phase of the optical carrier [1-2], but also facilitates near-ideal compensation of linear transmission impairments such as chromatic dispersion (CD) and polarization mode dispersion (PMD) [3]. In the coherent optical receivers, carrier frequency offset (CFO) estimation and compensation is essential to alleviate the requirement of the center wavelength alignment between the independently-running transmitter laser and the optical local oscillator (LO) [4-11].

For coherently detected quaternary phase-shift keying (QPSK) signals, several techniques have been proposed for blind CFO estimation. After data modulation removal by fourth power operation, the CFO estimation can be performed by either comparing the phase difference between adjacent samples [4], or finding the maximizing argument of the periodogram [5]. The data modulation can also be removed by modulus operation on the argument of the samples [6]. In addition, without data modulation removal, the CFO can be estimated by finding the frequency at which the amplitude of the discrete Fourier transform of the received samples achieves the maximum [7].

With the CFO estimation algorithm based on the periodogram of the fourth-powered samples, high estimation accuracy can be achieved [5]. However, it consumes considerable memory and computational resources due to fast Fourier transform (FFT). On the other hand, the CFO estimation algorithm based on comparing the phase difference between adjacent samples consumes much less memory and computational resources [4], but there is still considerable space to improve the estimation accuracy [8-9], if the phase difference between not only adjacent samples, but also samples with multiple time-domain separations is considered for the CFO estimation.

In this presentation, we investigate the utilization of the phase difference between samples with different time-domain separations to improve the CFO estimation accuracy. Similar concepts have been used in other applications, e.g. for differential phase-shift keying signals [12], or in self-coherent detection [13] to improve the receiver sensitivity. We show by simulation that in a 28-GBaud coherent QPSK system, this method can significantly reduce the standard deviation (STD) of the CFO estimation error by several orders of magnitude, when 16 time spans instead of one time span are used for the CFO estimation.

II. THE CFO ESTIMATION ALGORITHM AND SIMULATION SETUP

A. The CFO Estimation Algorithm

In a coherent optical QPSK receiver, the k^{th} received sample r_k after transmission impairment compensation is

$$r_k = A\exp\left\{ j\left(\frac{\pi}{2}d_k + 2k\pi f_c T + \varphi_k\right)\right\} + n_k \tag{1}$$

in which A is the amplitude of the received sample, $d_k \in \{0, 1, 2, 3\}$ is the k^{th} transmitted symbol, f_c is the CFO, T is the symbol duration time, φ_k is the carrier phase, and n_k is the additive Gaussian noise, respectively. For simple illustration of the principle we neglect the noise term as well as the difference between the carrier phases of adjacent samples; then the fourth-powered r_k multiplied by the conjugated fourth-powered r_{k-1} would be:

$$r_k^4\left(r_{k-1}^4\right)^* = |A|^8 \exp\{j8\pi f_c T\}. \tag{2}$$

Therefore, the CFO can be estimated as

$$\hat{f}_c = \frac{1}{8\pi T}\arg\left\{\sum_{k=1}^{N-1} r_k^4\left(r_{k-1}^4\right)^*\right\}, \tag{3}$$

in which N is the total number of samples utilized, and $\arg\{\bullet\}$ is the argument function. This is the estimator in [4]. In this presentation, we extend (3) by comparing the phase difference between samples with different time-domain separations, which is a positive integer denoted by m. Similar to (2), the fourth-powered r_k multiplied by the conjugated fourth-powered r_{k-m} is

$$r_k^4\left(r_{k-m}^4\right)^* = |A|^8\exp\{j8m\pi f_c T\}. \tag{4}$$

Therefore, for $m = 1, 2, \cdots, M$ we denote $\arg\left\{\sum_{k=m}^{N-1} r_k^4\left(r_{k-m}^4\right)^*\right\}$ as $\varphi(m)$, and unwrap $\varphi(1)$, $\varphi(2)$, $\cdots$, $\varphi(M)$ to get $\Phi(1)$, $\Phi(2)$, $\cdots$, $\Phi(M)$, then $\Phi(1)$, $\Phi(2)$, $\cdots$, $\Phi(M)$ would be

$$\begin{cases} \Phi(1) = 1\times 8\pi f_c T + \varepsilon_1 \\ \Phi(2) = 2\times 8\pi f_c T + \varepsilon_2 \\ \cdots \\ \Phi(M) = M\times 8\pi f_c T + \varepsilon_M \end{cases}, \tag{5}$$

in which ε_1, ε_2, $\cdots$, and ε_M are the random error terms. Based on this the CFO can be estimated by the least-squares method [14]

$$\hat{f}_c = \frac{\sum_{m=1}^{M} m\Phi(m)}{8\pi T\sum_{m=1}^{M} m^2}, \tag{6}$$

and in the following we will say the CFO estimation algorithm (6) utilizes M time spans.

B. Simulation Setup

We investigate the performance of the CFO estimation algorithm by simulation. At the transmitter side, a pseudorandom symbol sequence generated by MATLAB is mapped to QPSK constellation sequence at 28 GBaud, which is used to form an electrical square waveform. Then the electrical square waveform is passed through a Gaussian filter to adjust its rise/fall time to one fourth of the symbol duration time, and further smoothed by a third-order Bessel filter which has a 3-dB bandwidth of 56 GHz. Then the real and the imaginary parts of the electrical waveform are utilized to modulate a continuous-wave (CW) laser with an I/Q Mach-Zehnder modulator. The complex envelope of the CW laser has a constant amplitude, but its phase is modeled as a Wiener process to account for the laser linewidth. Additive white Gaussian noise is added to the optical signal to adjust its optical signal-to-noise ratio (OSNR). At the receiver side, the optical signal is passed through a 50-GHz second-order super-Gaussian optical bandpass filter and coherently detected. The optical-to-electrical converter and the following electrical circuits are noiseless and have fifth-order Bessel characteristics, with a 3-dB bandwidth of 24 GHz. Finally the electrical signal is sampled at the center of the symbols, the result of which is utilized to estimate the CFO. For determining the mean and the variance of the CFO estimation error, we run 1,000 rounds of simulation.

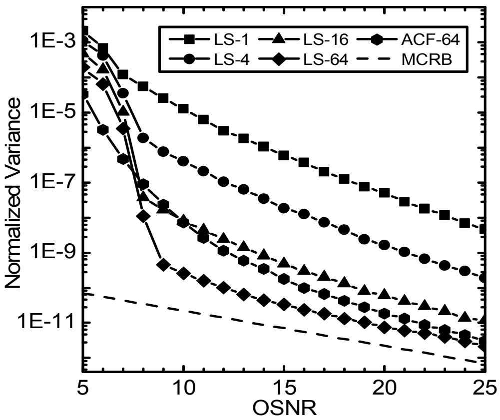

Figure 1. Normalized variance of the CFO estimation error as functions of OSNR with different CFO estimation algorithms, as well as the MCRB.

III. RESULTS AND DISCUSSIONS

Fig. 1 shows the variance of the CFO estimation error, normalized by multiplication with squared symbol duration time, as functions of OSNR for the least-squares algorithm (LS-1, LS-4, LS-16, and LS-64). LS-M corresponds to the algorithm in (6) that utilizes M time spans. For comparison, also plotted in Fig. 1 are that for the algorithm in [8] (ACF-64), as well as the modified Cramér-Rao bound (MCRB) – for all the algorithms that utilize N samples at signal-to-noise ratio of SNR – which is calculated as

$$\text{MCRB}(\hat{f}_c T) = \frac{3}{2\pi^2 N^3}\times\frac{1}{\text{SNR}}, \tag{7}$$

and serves as the theoretical limit [15]. The CFO is 2 GHz and 1,024 samples are utilized for the CFO estimation. Note that the variance of the CFO estimation error is inversely proportional to the number of samples utilized. Therefore, the ratio between the variances of the CFO estimation errors with two algorithms utilizing the same number of samples is the inverse ratio between the numbers of samples required for the two algorithms to achieve the same variance of CFO estimation error. If only one time span is used, the least-squares algorithm degenerates to the algorithm in [4]. The performance is far from the MCRB and the ratio between the variance of the CFO estimation error $\sigma^2_{\text{LS-1}}$ and the MCRB σ^2_{MCRB} is more than 2.7×10^5 at 11.5-dB OSNR. In the cases that more time spans are utilized, the variances $\sigma^2_{\text{LS-4}}$, $\sigma^2_{\text{LS-16}}$, and $\sigma^2_{\text{LS-64}}$ are still high at low OSNR values. But they drop dramatically when the OSNR increases from 6 dB to 9 dB and the ratios between the variances and σ^2_{MCRB} stabilize as the OSNR further increases. At 11.5-dB OSNR the ratio between $\sigma^2_{\text{LS-64}}$ and σ^2_{MCRB} is less than

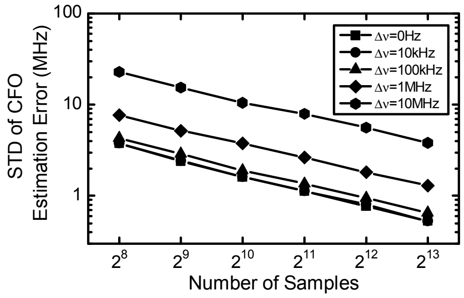

Figure 2. The STD of the CFO estimation error as functions of number of samples utilized for CFO estimation when the laser linewidth is 0 Hz, 10 kHz, 100 kHz, 1 MHz, and 10 MHz.

8.2. Compared with ACF-64 that utilizes the same number of time spans, the variance $\sigma^2_{\text{LS-64}}$ is higher at low OSNR. However, the ratio of $\sigma^2_{\text{LS-64}}$ over $\sigma^2_{\text{ACF-64}}$ decreases rapidly when the OSNR increases from 6 dB to 9 dB, after which the ratio begins to increase. Nevertheless, $\sigma^2_{\text{LS-64}}$ associated with the least-squares method is always lower than $\sigma^2_{\text{ACF-64}}$ when the OSNR is greater than 8 dB. At 11.5-dB OSNR the ratio of $\sigma^2_{\text{LS-64}}$ over $\sigma^2_{\text{ACF-64}}$ is less than 0.073, implying that to achieve the same estimation accuracy as LS-64, ACF-64 requires a sample number 13.7 times as many as that for LS-64. This verifies the advantages of the least-squares method over conventional methods [4, 8].

Fig. 2 shows the STD of the CFO estimation error as functions of number of samples utilized for CFO estimation, when the linewidth of the transmitter laser and the optical LO $\Delta\nu$ is 0 Hz, 10 kHz, 100 kHz, 1 MHz, and 10 MHz, respectively. It is assumed that the CFO is 2 GHz, the OSNR is 11.5 dB, and 16 time spans are utilized for the CFO estimation. The STD of the CFO estimation error reduces by half when the number of samples increases by a factor of four. When the laser linewidth is 10 kHz, the STD of the CFO estimation error is nearly identical to the ideal case where the laser linewidth is zero. When the laser linewidth increases to 100 kHz, the STD of the CFO estimation error increases only by less than 25%. When the laser linewidth further increases to 1 MHz, however, the STD of the CFO estimation error could increase significantly by 150%. Since the linewidth of the lasers in coherent communication systems is commonly <200 kHz, it is expected that the least-squares CFO estimation algorithm can realize optimal operation with negligible performance penalty for practical lasers.

We also investigate the effect of the residual transmission impairments on the performance of the least-squares CFO estimation algorithm which utilizes 16 time spans and 512 samples. The linewidth of the transmitter laser and the optical LO is 100 kHz, the CFO is fixed at 2 GHz, and the OSNR is 11.5 dB, respectively. Fig. 3(a) and Fig. 3(b) show the estimated mean and its 95% confidence interval, and the STD of the CFO estimation error versus CD, respectively. When the residual CD is between -140 ps/nm and 140 ps/nm, the estimated mean of the CFO estimation error is between -0.2 MHz and 0.2 MHz, and

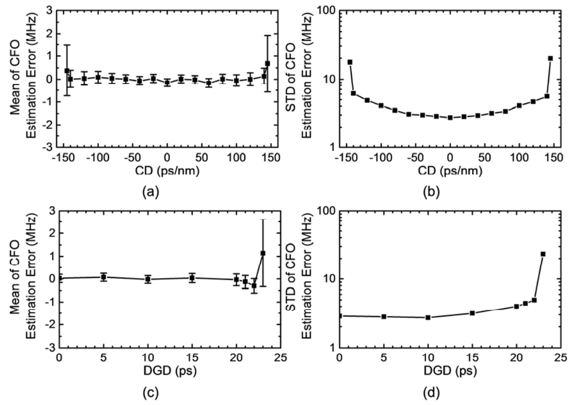

Figure 3. (a) The mean and its 95% confidence interval and (b) the STD of the CFO estimation error as functions of CD; (c) the mean and its 95% confidence interval and (d) the STD of the CFO estimation error as functions of DGD.

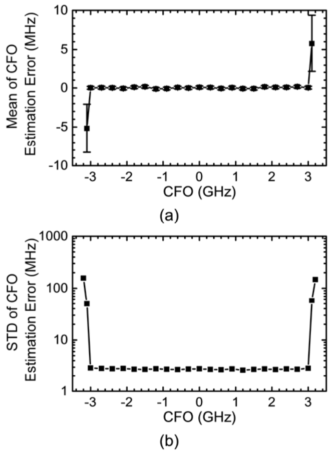

Figure 4. (a) The mean and its 95% confidence interval and (b) the STD of the CFO estimation error as functions of the CFO.

the estimated STD of the CFO estimation error is less than 6.2 MHz. After CFO compensation, the residual CFO is within the tolerance of most carrier phase estimation algorithms. Fig. 3(c) and Fig. 3(d) show the estimated mean and its 95% confidence interval, and the STD of the CFO estimation error versus differential group delay (DGD), respectively. The STD of the CFO estimation error can be maintained within 5 MHz for the DGD values less than 22 ps. In practice, the residual DGD after PMD compensation is well below this value, so the least-squares algorithm can be combined with dispersion compensation algorithms to realize optimal system performance.

Fig. 4 shows the mean and the STD of the CFO estimation error of the least-squares algorithm as functions of the CFO. The CFO estimation algorithm utilizes 16 time spans and 512 samples. The linewidth of the transmitter laser and the optical LO is 100 kHz, and the OSNR is 11.5 dB, respectively. The variations of the mean and the STD of the CFO estimation error are very small when the CFO is between -3 GHz and 3 GHz. The mean of the CFO estimation error is between -0.2 MHz and 0.2 MHz, and the STD of the CFO estimation error is less than 3 MHz. As the CFO increases beyond 3 GHz, the CFO estimation error rises abruptly. Therefore, the estimation range of the least-squares CFO estimation algorithm is [-3GHz, 3GHz], the span of which is around one fourth of the baud rate. In practice, the CFO between the independently-running transmitter laser and the optical LO can be readily controlled within this range.

IV. Conclusions

For coherent optical QPSK receivers we have investigated a computationally efficient least-squares CFO estimation algorithm that utilizes the phase difference between samples with different time-domain separations. Simulation results show that the estimation accuracy increases rapidly as the number of time spans utilized for CFO estimation increases. When 16 time spans and 512 samples are utilized for CFO estimation, at 11.5-dB OSNR, the STD of the CFO estimation error of the least-squares algorithm for 28-GBaud QPSK signal is less than 3 MHz when the CFO varies between -3 GHz and 3 GHz. The CFO estimation algorithm can be implemented in burst-mode coherent optical QPSK receivers in which efficiency in terms of both the number of samples and the hardware resources required is very important.

Acknowledgment

This work was supported in part by Hong Kong Research Grants Council under Direct Grant for Research, the University Grant Committee of HKSAR under Grant number AoE/E-02/08, the Science Foundation Ireland under grant number 11/SIRG/I2124 and 13/TIDA/I2718, and Irish Research Council New Foundations 2013 CDAMT.

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