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Exact Analysis of Homodyne Crosstalk Induced Penalty in Optical Networks

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Abstract:

Homodyne crosstalk causes severe system performance degradation in optical networks by beating with the desire signal. While Gaussian approximation overestimates the performance degradation, for a single dominant crosstalk source, the exact noise probability distribution and closed-form error probability is derived in this paper.

Keywords: crosstalk interference, optical networks, WDM systems.

I. INTRODUCTION

The basis of future information infrastructure will be built upon on multi-wavelength optical networks in which channel routing and add-drop functions are performed by wavelength routers. A fundamental difficulty of wavelength router is the homodyne crosstalk from neighboring inputs, causing severe degradation in system performance. Having similar or identical wavelength to that of the signal, this crosstalk is difficult to be eliminated by filtering, which will beat with the signal and generate a new kind of noise at the receiver [1]-[7]. Homodyne crosstalk induces higher penalty then crosstalk from other wavelength due to signal beating.

Representing a worst-case assumption and serving well for conservative system design [4, 6, 7], Gaussian approximation was used in previous studies on wavelength router homodyne crosstalk [1, 3-6], though there were reports and evidences that the Gaussian assumption is questionable for smaller number of crosstalk sources [4, 7, 8]. From the central-limit theorem, the Gaussian assumption is convincing for the combination of a large number of more or less identical and independent interference sources [3]. But in most cases, the number of dominant interference source is limited to one or two due to near-far effect in the optical network. Therefore, a non-Gaussian model may estimate the system performance more accurately. Here, we provide an exact analysis for a single dominant homodyne crosstalk source. In later part of this paper, a closed-form bit-error-rate (BER) formula is also provided. The results are in good agreement with that obtained by experiments [6].

II. ANALYSIS OF HOMODYNE CROSSTALK

Fig. 1 shows an example of an optical network that may induce homodyne crosstalk with a similar or identical wavelength to the signal wavelength. While the channel at wavelength λ_i at the input of wavelength router 1 should not appear at the input of wavelength router 2, due to insufficient crosstalk rejection in router 1, small amount of crosstalk appears at the input of wavelength router 2 as homodyne crosstalk. This homodyne crosstalk will beat with the signal wavelength and severely degrade the system performance. While crosstalk from other wavelength is proportional to the square of the crosstalk electric field, homodyne crosstalk is proportional to the multiplication of the signal and crosstalk electric field. Usually, with the same crosstalk intensity, the effects homodyne crosstalk is usually one order larger than that caused by crosstalk from other wavelengths.

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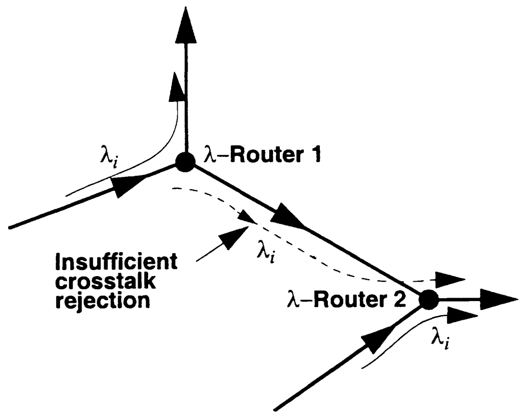

Fig. 1. Example of an optical network configuration that may induce homodyne crosstalk.

For simplicity, first assuming an optical signal without modulation, the electrical intensity of the desired optical signal is $E_i(t) = E_i e^{-j\omega_i t - j\varphi_0(t)}$, and a small homodyne crosstalk originated from a neighboring node is $E_{i,x}(t) = \sqrt{x} E_i e^{-j\omega_i t - j\varphi_1(t)}$, where x is the crosstalk level in optical power, ω_i is the angular frequency of the optical signal, and $\varphi_0(t)$ and $\varphi_1(t)$ are the random phase due to laser phase fluctuation.

Without loss of generality, for a unit detector responsivity and identical polarization, the photocurrent is

$$i(t) = \left|E_i + \sqrt{x} E_i e^{-j\varphi(t)}\right|^2, \quad (1)$$

where $\varphi(t) = \varphi_1(t) - \varphi_0(t)$ is also a random phase. Ignoring the small term in order of x first, the overall receiver noise in the photo-detector is [1]-[6],

$$n(t) = A\cos[\varphi(t)] + n_0(t), \quad (2)$$

where $A = 2\sqrt{x}\, E_i^2$ is the crosstalk amplitude, and $n_0(t)$ is the usual Gaussian noise in the receiver. To calculate the BER, the probability density function (p.d.f) of $n(t)$ has to be evaluated.

The p.d.f. of $A\cos[\varphi(t)]$ is given by $p(x) = (1/\pi)(A^2 - x^2)^{1/2}$ for $-A < x < +A$ [4, 7-8] which yields the characteristic function of $J_0(A\omega)$, where $J_0(\bullet)$ is the Bessel function. The characteristic function of $n(t)$ then becomes

$$\psi_n(\omega) = J_0(A\omega)\exp(-\sigma^2\omega^2/2), \quad (3)$$

where σ^2 and $\exp(-\sigma^2\omega^2/2)$ are the variance and the characteristic function of the receiver Gaussian noise $n_0(t)$, respectively. Taking a series expansion and found the inverse Fourier transfer of the characteristic function, the p.d.f. of $n(t)$ is [9] [10, §9.3]

$$p_n(r) = \frac{1}{\sqrt{2\pi}\sigma} \sum_{k=0}^{\infty} \frac{(-r^2/2\sigma^2)^k}{k!}\ {}_1F_1\left(k + \frac{1}{2};\ 1\ ;\ -\frac{A^2}{2\sigma^2}\right), \quad (4)$$

or

$$p_n(r) = \frac{1}{\sqrt{2\pi}\sigma}\sum_{k=0}^{\infty}\frac{(A^2/2\sigma^2)^k}{2^k(k!)^2}H_{2k}\left(\frac{r}{\sigma}\right)\exp\left(-\frac{r^2}{2\sigma^2}\right), \tag{5}$$

where $A^2/2\sigma^2$ is the ratio of crosstalk to Gaussian variance, ${}_1F_1(a;\, b;\, x)$ is the confluent hypergeometric function [10, §A.1.2], and

$$H_n(x) = (-1)^n e^{x^2/2}\frac{d^n e^{-x^2/2}}{dx^n} \tag{6}$$

is a Hermitian polynomial of order n. The p.d.f. provided by (4) is useful when r/σ is reasonably small, while (5) is convenient for large r/σ or small A/σ.

Assuming a detection level of d, the error probability is

$$p_b = \frac{1}{2} - \frac{1}{\sqrt{2\pi}}\sum_{k=0}^{\infty}\frac{(-1)^k(d/\sigma)^{2k+1}}{2^k(2k+1)k!}\ {}_1F_1\left(k+\frac{1}{2};\, 1\, ;\, -\frac{A^2}{2\sigma^2}\right), \tag{7}$$

or

$$p_b = \frac{1}{2}\mathrm{erfc}\left(\frac{d}{\sqrt{2}\sigma}\right) + \frac{1}{\sqrt{2\pi}}\sum_{k=1}^{\infty}\frac{(A^2/2\sigma^2)^k}{2^k(k!)^2}H_{2k-1}\left(\frac{d}{\sigma}\right)\exp\left(-\frac{d^2}{2\sigma^2}\right). \tag{8}$$

The BER at the receiver can be evaluated according to the error probability p_b.

The BER of the system is $Q(\rho_G)$ for an optical communication system contaminated by only Gaussian noise, where $Q(x) = 1/\sqrt{2\pi}\int_x^{\infty}e^{-x^2/2}dx$ is the cumulative tail of the normalized Gaussian distribution, $\rho_G = (I_1 - I_0)/(\sigma_1 + \sigma_0)$ is the Q-factor or signal-to-noise (SNR) ratio of the system, and I_1, I_0, ${\sigma_1}^2$, ${\sigma_0}^2$ are the photocurrent and the noise variance at ONE and ZERO levels, respectively. While the threshold level can be optimized in the laboratory for better performance, for field applications, the threshold is usually set to the middle of the "eye" [8]. For an infinite extinction ratio (or I_1/I_0 larger than 20 dB), and identical ZERO and ONE level Gaussian noise, the threshold level $d = I_1/2$ and $\sigma = \sigma_1 = \sigma_0$, the Q-factor $\rho_G = d/\sigma$.

In (7) and (8), $A^2/2\sigma^2$ is equal to the ratio of crosstalk to Gaussian noise. The homodyne crosstalk occurs only if both the signal and crosstalk channel are transmitted in ONE levels, or $A = 2\sqrt{2d}\sqrt{2dx}$. After some algebra, we can find that $A^2/2\sigma^2 = 8\rho_G^2 x$. The threshold at the middle of the "eye" is $(1 + x)I_1/2$. Considering all combination of ONE and ZERO levels of the signal and crosstalk channel, with the term of x also taking into account, the closed-form BER formula corresponding to (8) is:

$$\begin{aligned}\mathrm{BER} &= \frac{1}{2}Q[\rho_G(1-x)] + \frac{1}{2}Q[\rho_G(1+x)] \\ &+ \frac{1}{4\sqrt{2\pi}}\sum_{k=1}^{\infty}\frac{2^{2k}\rho_G^{2k}x^k}{(k!)^2}H_{2k-1}[\rho_G(1+x)]\exp\left[-\frac{\rho_G^2(1+x)^2}{2}\right]\end{aligned}, \tag{9}$$

where the first two terms are contributed by Gaussian noise, the third term is contributed by the non-Gaussian nature of homodyne crosstalk. Similar formula corresponding to (7) can also be derived. The first-term in (9) is for signal and crosstalk channel in different levels. The second term in (9) is the effect of

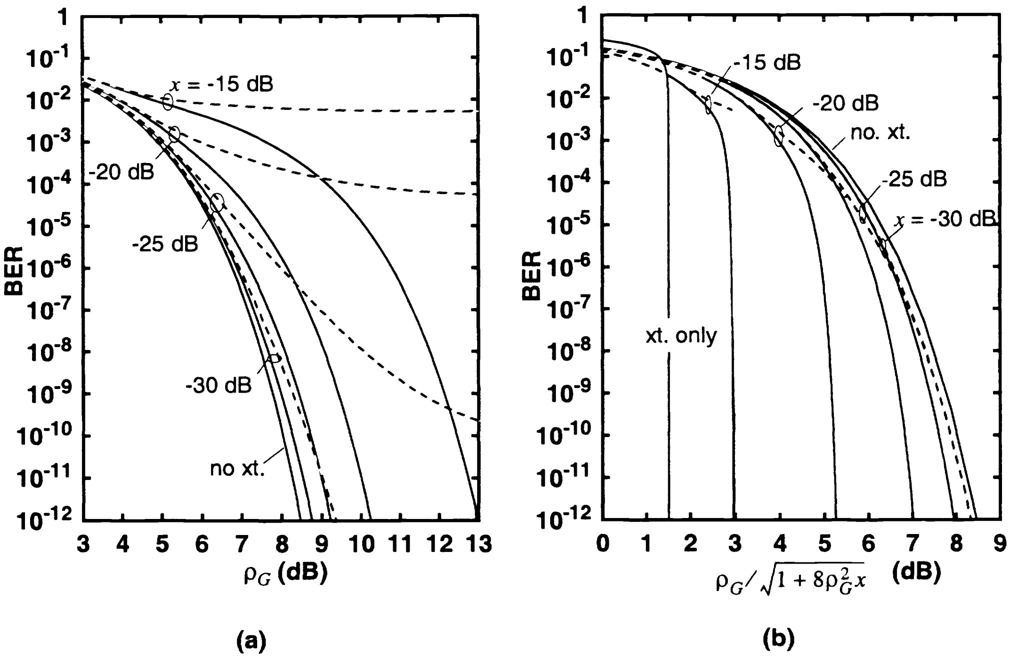

Fig. 2. BER as a function of SNR for different crosstalk levels. SNR represented by both ρ_G and $\rho_G/\sqrt{1+8\rho_G^2 x}$ are shown for comparison.

Gaussian noise for both crosstalk and signal channel in the same level. The third term in (9) is the effect of crosstalk for both crosstalk and signal channel in ONE level.

With Gaussian assumption, the BER can be approximated by

$$\mathrm{BER} \approx \frac{1}{2}Q[\rho_G(1-x)] + \frac{1}{4}Q[\rho_G(1+x)] + \frac{1}{4}Q\left[\frac{\rho_G(1+x)}{\sqrt{1+8\rho_G^2 x}}\right]. \quad \textbf{(10)}$$

Usually, the BER in (10) is dominated by the last term. Note that $8\rho_G^2 x$ is the crosstalk to Gaussian noise ratio for both signal and crosstalk channel in ONE levels, the average ratio is $4\rho_G^2 x$ [1, 3, 8].

Fig. 2a and Fig. 2b shows BER as a function of SNR of ρ_G and $\rho_G/\sqrt{1+8\rho_G^2 x}$, respectively. The SNR of $\rho_G/\sqrt{1+8\rho_G^2 x}$ is the dominated term in (10) which represents the signal-to-overall noise ratio. Fig. 2 shows that Gaussian approximation overestimates the degradation induced by homodyne crosstalk. While Gaussian approximation (10) is valid in the range of small SNR, the BER provided by Gaussian approximation (10) is always higher than the BER provided by e exact analysis (9). For example, the Gaussian approximation provides a BER floor around 10^{-2} for a crosstalk level of -15 dB but exact analysis shows no BER floor.

Without crosstalk, the SNR of the system to achieve a BER = 10^{-9} is $\rho_G = 6$. The system crosstalk penalty then equals to $10\log_{10}(\rho_G/6)$, where ρ_G is the required SNR for BER = 10^{-9} with homodyne crosstalk. Fig. 3 shows crosstalk penalty as a function of crosstalk ratio for system with negligible input power depending noise, for example, system with only thermal noise. The power penalty derived from Gaussian approximation [1, 8] is also shown as comparison.

Our theoretical results are verified by experiments using the setup shown in Fig. 4. A DFB laser is modulated by a 622-Mb/s (2^{15}-1 PBRS) NRZ data with an extinction ratio of 15 dB. To simulate homodyne crosstalk, the modulated signal is split into two paths, with one path 7 km longer than the other to avoid coherent effects. The polarization of the crosstalk channel is carefully controlled to yield maximum beat noise at the receiver. The measured crosstalk-induced system penalties are in excellent agreement with that obtained from the analysis.

III. CONCLUSION

In optical networks, severe system performance degradation is induced by homodyne crosstalk with identical wavelength to the signal channel. Conventionally, even for a single interference source, Gaussian approximation is used to estimate the BER performance for a conservative system design. An exact analysis and a closed-form BER formula of homodyne crosstalk is provided here for a single crosstalk source.

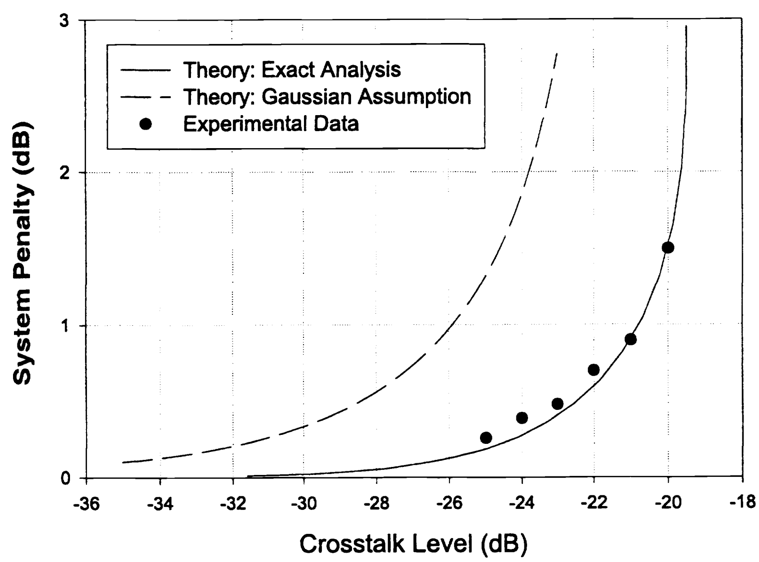

Fig. 3. System penalty as function of crosstalk level.

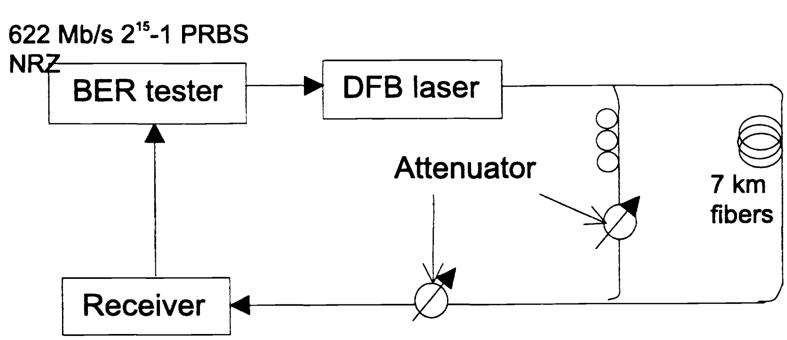

Fig. 4. Experimental setup to measure homodyne crosstalk induced power penalty.

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