Hard-Decoding Vector Quantization Using Bayes Estimator for Rayleigh Channel

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Abstract:

Vector quantization is decoded using Bayes estimator. Hard-decision Bayes estimator is used to reduce the complexity involving the evaluation of transition probabilities for soft-decision Bayes estimator. In the range of low to moderate average channel signal-to-noise ratio, the hard-decision Bayes estimator based source decoder can achieve more than 1 dB improvement over conventional mapping based source decoder. With great reduction in complexity, the hard-decision Bayes estimator can achieve a performance that is just about 0.4 dB below the optimal soft-decision Bayes estimator.

1. Introduction

Vector quantization is an important method for data compression and source encoding. It is for great importance that vector quantizers are robust against channel noise. Traditionally, source and channel codes are designed separately and then cascaded together using a mapping based source decoder. Provided with an infinite degree of complexity and delay, source and channel coding can be performed separately without sacrificing fidelity [1]. In practical system, the channel decoder can provide some information to the source decoder in which the objective is to minimize the overall distortion from the source at the transmitter to the reconstruction at the receiver.

Most studies of combined source-channel coding lead to the study of channel-optimized scalar quantizers or channel-optimized vector quantizers (COVQ) for noisy channel [2]-[6]. The design algorithms of COVQ optimize the quantizer under the assumption of a noisy channel, usually a binary symmetric channel (BSC) that has a fixed error probability. Even for an additive noise channel with well-known statistics, those algorithms [2]-[6] simplify the channel to BSC by hard-decision decoding. In COVQ, the channel bit-error rate (BER) is required in the transmitter to design the encoder. While the BER can be measured easily at the receiver end, its availability at the transmitter may not easily be guaranteed. An encoder is also very difficult to optimize for many receivers for a broadcasting system. Furthermore, a COVQ must be designed in real time or many stored encoders for various BERs must be stored and retrieved in real implementation.

Another approach to reduce the overall distortion is to use soft decoding for robust vector quantization [7]-[13]. Usually, a source-optimized vector quantizer (SOVQ) for a noiseless channel is first designed and the decoder soft-decodes the signal according to Bayes estimation [7]-[9], Hadamard matrix [10]-[13], log-likelihood ratio [14], or a suboptimal linear receiver [15]. Those soft decoders achieve great improvement over mapping based decoder designed by the separation principle. However, the decoder complexity is high due to the requirement of frequent calculation of some transcendent function [7]-[14].

In addition to the soft-decision Bayes estimator, this paper purposes the application of hard-decision Bayes estimator as a vector quantization decoder. In the hard-decision Bayes estimator, the channel decoder just requires to provide the BER value to the source decoder. In other words, the hard-decision Bayes estimator treats the channel as BSC (may be time-varying) instead of a continuous channel. The source decoder also just requires to make simple arithmetic calculation. The hard-decision Bayes estimator based source decoder can operate in all channels as long as the BER can be measured. As an application to wireless communications, the hard-decision Bayes estimator is studied for Rayleigh flat-fading channels.

2. Hard-Decision Bayes Estimator

Fig. 1 shows schematic diagrams of a communication system for an analog discrete-time source using either hard- or soft-decision Bayes estimator as source decoder. The source is quantized by either a scalar or vector quantizer to codeword at the trans-

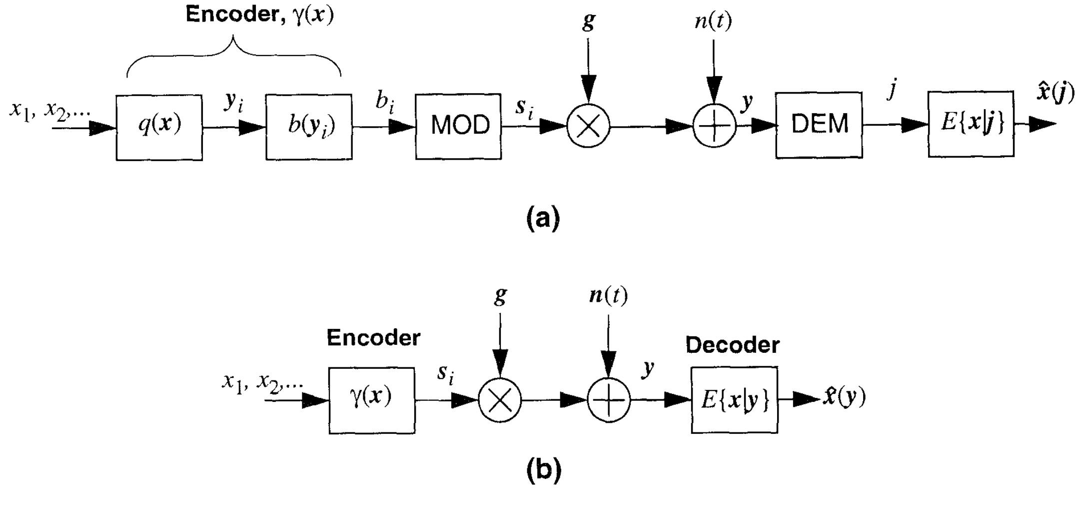

Fig. 1 Schematic diagram of a system to transmit analog source using (a) hard-decision and (b) soft-decision Bayes estimator as source decoder.

mitter. The codeword is mapped to a binary codeword for a digital modulator. For simplicity, assuming baseband, BPSK, or QPSK modulation, the received signal at the receiver is

$$y = g \cdot s_i + n, \tag{1}$$

where g is a random variable representing the channel fading, $s_i = (b_{i1}, \ldots, b_{iB})$, $i = 1,\ldots, Q$, $b_{ik} = \pm 1$, is the modulated signal, and n is channel additive noise, B is the number of bits per symbol and Q is the number of quantization levels. For a fading channel without interleaving or with symbol-wise interleaving, the variable g is a scalar random variable. The variable $g = (g_1,\ldots, g_B)$ is a vector random variable with bit-wise interleaving and, not a standard notation, $g \cdot s_i$ denotes element-wise multiplication $(b_{i1}g_1,\ldots, b_{iB}g_B)$.

Using soft-decoding, for minimum mean-squared error [16], the decoder estimates the source vector by

$$\hat{x}(y) = E\{x|y\} = \sum_{i=1}^{Q} c_i p(s_i|y), \tag{2}$$

where c_i, $i = 1,\ldots, Q$, are the codewords and centroids of each partition and $p(s_i|y)$, $i = 1,\ldots, Q$, are the transition probabilities. From Bayes rule,

$$p(s_i|y) = \frac{p(y|s_i, g)p_{s_i}}{\sum_{i=1}^{Q} p(y|s_i, g)p_{s_i}} \tag{3}$$

where p_{s_i} is the *a priori* probability of the symbol s_i. The soft decoder and its variations can achieved significant improvement over the conventional mapping based source decoder [7]-[14]. However, the soft decoder requires high complexity for all the calculation to evaluate $p(s_i|y)$. While the performance may not be as good as soft decoder, a hard-decision Bayes estimator is discussed in this paper with performance close to that of soft decoder.

The receiver first demodulates the input y into a binary codeword j when a hard-decision Bayes estimator is used. The decoder estimates the source vector by

$$\hat{x}(j) = E\{x|j\} = \sum_{i=1}^{Q} c_i p(s_i|j). \tag{4}$$

From the Bayes rule,

$$\hat{x}(j) = \frac{\sum_{i=1}^{Q} c_i p(j|s_i, g)p_{s_i}}{\sum_{i=1}^{Q} p(j|s_i, g)p_{s_i}}, \tag{5}$$

where $p(j|s_i, g)$ are the transition probabilities from symbol s_i to symbol j. If all bits have the same BER, the transition probabilities are

$$p(j|s_i, g) = p^{d(i,j)}(1-p)^{B-d(i,j)}, \tag{6}$$

where p is the BER as a function of $\mathbf{g}$ and $d(i, j)$ is the Hamming distance between i and j. If all bits have different BER, the transition probabilities are

$$p(j|s_i, \mathbf{g}) = \prod_{k=1}^{B} p_k^{l_k(i,j)}(1-p_k)^{1-l_k(i,j)} , \quad (7)$$

where $l_k(i, j) = 0$ if the kth bit of i and j are identical and $l_k(i, j) = 1$ if the kth bit of i and j are different, p_k is the BER of the kth bit. The real-time BER p_k can be measured by various methods, for example, that used in SONET system [19] with some overhead. Alternatively, for additive Gaussian noise, $p_k = Q(g_k/\sigma)$ where σ is the standard deviation of the Gaussian noise. Even the noise characteristic of the channel is unknown, the BER can be monitored.

In the hard-decision Bayes estimator, the transition probabilities (7) can be approximated as

$$p(j|s_i, \mathbf{g}) = \begin{cases} 1 & i = j \\ p_k & l_k(i, j) = 1 \\ 0 & d(i, j) \geq 2 \end{cases} \quad (8)$$

to reduce the computational complexity. The approximation takes into account only cases of error-free and single-bit error. The probabilities of multiple-bit error are assumed to be zero. In the approximation, almost no calculation is required to evaluate the transition probabilities. Although greatly reducing the complexity of the calculation, this simplification of the transition probabilities will induce some additional distortion. As shown later, in the usual cases, the additional degradation is insignificant.

3. Numerical Results

The hard-decision Bayes estimator is applied for Rayleigh flat-fading channel. Rayleigh channel is assumed because the scheme may be more suitable for wireless communication system experienced with power fading and high bit error. For Rayleigh channel, assuming the complicate bit-wise interleaving, the vector random variable $\mathbf{g}$ has a joint probability density function of

$$p_G(\mathbf{g}) = \prod_{k=1}^{B} g_k e^{-g_k^2/2} , g_k > 0. \quad (9)$$

For soft decoder, assuming additive Gaussian noise, the general expression of the continuous transition probabilities of $p(\mathbf{y}|s_i, \mathbf{g})$ is:

$$p(\mathbf{y}|s_i, \mathbf{g}) = \frac{1}{(2\pi\sigma^2)^{B/2}} \prod_{k=1}^{B} \exp\left(-\frac{(y_k - g_k b_{ik})^2}{2\sigma^2}\right) \quad (10)$$

In order to calculate these transition probabilities, many exponential function evaluations are required in the decoder of (2) and (3).

Using the hard-decision Bayes estimator in the Rayleigh flat-fading channel, only one BER evaluation or measurement is required within the coherence time of the channel. For bit-wise interleaving, the BER of each bit may be stored and "de-interleaved" with the bit. However, even for many bits, one BER value is required for all bits within the channel coherence time. With symbol-wise interleaving, the vectors of $\hat{x}(j)$ can also be stored and used within the coherence time. The decoder using the approximated transition probabilities (8) can also be used for decoding.

The source is a first-order Gauss-Markov random sequence represented by

$$x_i = \rho x_{i-1} + e_i , \quad (11)$$

where e_i is sequence of independent identically distributed Gaussian random variables with zero mean and unit variance. The correlation between successive samples, ρ, is set to 0.9.

Fig. 2 shows the signal-to-distortion ratio (S/D) as a function of the average channel signal-to-noise ratio (S/N) for a Rayleigh flat-fading channel. The average channel S/N is calculated by $E\{\mathbf{g}^2\}/E\{\mathbf{n}^2\}$ and the S/D is evaluated by $E\{\mathbf{x}^2\}/E\{(\mathbf{x}-\hat{\mathbf{x}})^2\}$. The source is quantized by a

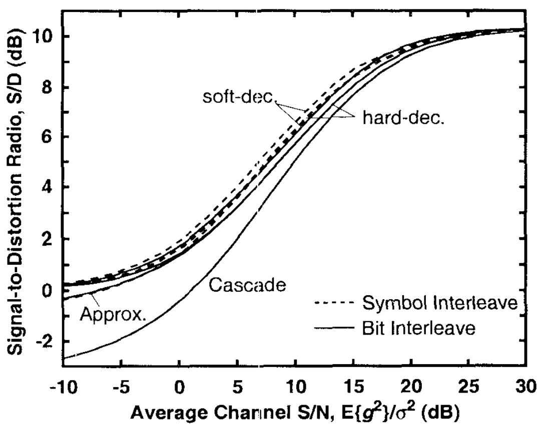

Fig. 2 Signal-to-distortion ratio (S/D) as a function of average channel SNR for a four-dimensional vector quantizer using both hard- and soft-decision Bayes estimator as source decoder.

four-dimensional vector quantizer with a rate of 1 bit/sample. The vector quantizer is a SOVQ trained by the LBG algorithm with splitting codeword initialization [17]-[18].

The performance of both hard- and soft-decision source decoder is shown for comparison for both symbol- and bit-wise interleaving with both exact (7) and approximated (8) transition probabilities. In general, the symbol-wise interleaving performs better than bit-wise interleaving. However, bit-wise interleaving provides smaller variation in S/D over time but symbol-wise interleaving provides larger variation in S/D over time.

All schemes have the same S/D at high channel S/N because the channel has small additive noise and negligible channel induced distortion. At very low channel S/N, the conventional mapping based decoder for vector quantizer is equivalent to randomly choose another vector to represent the original vector. Therefore, the squared-distortion is approximately twice the variance of the original signal and the S/D is approximately -3 dB. Other schemes except that using approximated transition probabilities provide a reconstruction level of zero vector and the S/D approaches 0 dB at very low channel S/N.

Using either hard- and soft-decision Bayes estimator, the S/D improves in most levels of average channel S/N. Compared with conventional cascaded source and channel coding, the improvements of hard-decision Bayes estimator with symbol- and bit-wise interleaving is larger than 1 dB for an average channel S/N less than about 12 and 7 dB, respectively. The improvement is larger for small S/N and approaches the asymptotic 3 dB improvement at extremely low S/N. With significant reduction in complexity compared to soft decoder, the hard-decision Bayes estimator has a maximum S/D reduction of about 0.4 dB.

The hard-decision Bayes estimator using approximated transition probabilities always performs worse then that using exact transition probabilities. However, in the range of average channel S/N larger 0 dB, the Bayes estimator using the approximated transition probabilities has a performance having insignificant difference with that using the exact transition probabilities. Since no calculation is required to evaluate the approximated transition probabilities, it may be a good choice in practical implementation.

4. Conclusion

Hard-decision Bayes estimator based source decoder is proposed as decoder of vector quantization for noisy channel. Instead of using a mapping based conventional decoder designed on the separation principle, provided the instantaneous channel BER, the source decoder estimates the expected source vector given the hard-decision symbol. Compared with soft-decision Bayes estimator, the hard-decision Bayes estimator provides great complexity reduction, especially that using approximated transition probabilities. Reducing the channel to a time-varying BSC, the hard-decision Bayes estimator can be used in all channels as long as the instantaneous BER can be measured or evaluated.

Numerical results are presented for a four-dimensional vector quantizer transmitted through a Rayleigh flat-fading channel. The hard-decision Bayes estimator based source decoder achieves great improvement to the cascaded decoder, especially at low average channel S/N. The improvement is larger than 1 dB for an average channel S/N less than about 12 and 7 dB for symbol- and bit-wise interleaving, respectively.

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