Resource Optimization of Consolidating Two Coexisting Networks With Interconnections

Zhenchang Xie and Lian-Kuan Chen

***Abstract*—We investigate the consolidation of two optical networks, which overlap in some geographical areas, by installing interconnection links at strategic locations to reduce system operational costs. We focus on the operational costs of fiber links in the networks. The minimum number of operational fiber links required to provide bi-directional connectivity for any two nodes of the merged network is examined. Analytical results show that it is at least the number of nodes or at least twice the number of bridges when all the co-located nodes are interconnected. The optimal locations of interconnection links are derived when the interconnection cost is very high, which results in a minimum of two interconnections. They should be installed at the nodes which are one hop away from two certain cut nodes if there are cut nodes in the networks. To take into account more practical considerations, single link failure protection schemes for the merged network are also studied.**

***Index Terms*—Fiber optics links and subsystems; Networks; Network optimization.**

I. Introduction

Telecommunication infrastructure has been growing rapidly in the last few decades [1]. Free competition and deregulation propel carriers into the massive construction of separate transport networks to compete with each other. The network infrastructures may overlap extensively in some regions. Network resources are not optimized and are perhaps wasted due to the coexistence of multiple networks by different carriers [2,3]. After telecommunications deregulation, strategic alliances, mergers and acquisitions of competitive operators are often seen, and they lead to the consolidation of multiple networks. In recent years, consolidation has been extended to the convergence of networks to provide triple play services, namely, voice, video, and data services [4–7]. This may lead to infrastructure consolidation of telecommunication networks and cable TV networks. With the successful demonstration of single channel over 100 Gb/s non-repeated transmission systems using various spectral-efficient modulation formats such as quadrature phase-shift keying and quadrature amplitude modulation, the number of operational fiber links required can be reduced to achieve higher utilization of network resources [8,9]. In the consolidation of two networks, through traffic grooming and rerouting [10–14], some of the links can be suspended. Although the installed fibers of the suspended links cannot be reallocated, the operation cost of the regenerator sites can be reduced [15,16]. Cost savings can further be made from the reduced power consumption and efficient utilization of the network resources at the nodes such as optical cross connects (OXCs) and transceivers [17]. Also, the interconnectivity between the two networks can be supported more smoothly.

We will investigate the consolidation of two optical networks which overlap at some regions by installing interconnection links at certain strategic locations to achieve the most savings in the number of operational fiber links. The non-overlapped areas are not of much interest for investigation since they can be optimized independently. We focus on the overlapping areas; thus the two networks are seen as if they are identical in those areas. It is worth noting that, when the construction cost of the interconnection links varies, the optimal number of interconnection links to be employed for the consolidation will also vary. Thus the variation of the interconnection build cost will result in different required numbers of operational fiber links. To simplify the problem and derive some insightful analytical results for the consolidation of two optical networks, two extreme scenarios of network merger will be discussed. First, we will consider negligible cost for the construction of interconnection links such that all the co-located nodes can have interconnection links installed. We will derive the minimum number of operational fiber links required to provide bi-directional connections between any two nodes of the two networks in this case. Very high installation cost of the interconnection links is assumed for the second case. We will prove in this case that only two interconnection links should be installed, and their optimal locations will be given. The operational fiber links in the merged network will also be identified. Single link failure protection schemes for 1:1 protection of the merged network for the two cases will also be discussed.

The rest of the paper is organized as follows. Section II presents the formulation of the problem with some basic assumptions. Two cases of network merger under negligible interconnection cost and very high interconnection cost will be discussed in Sections III and IV, respectively. In Section V, we consider the connectivity efficiency of different network topologies for both cases. Section VI discusses the link failure protection issues of the merged network. Section VII concludes this paper.

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II. Assumptions and Problem Formulation

A. Assumptions

We consider only connected networks, and model the existing optical networks as directed planar graphs. This means that there are two separate links of opposite directions connecting two nodes if there is a physical fiber link between these two nodes. It is assumed that there are two identical network parts in the two optical networks such that their nodes and fiber links are all co-located, and the operation cost of every fiber link is identical [18]. We are optimizing the identical areas of the two networks as there are more redundant links. In the following discussions, the two identical networks refer to the identical network parts. For the non-overlapping parts, they can be optimized independently and will not be discussed in this paper. We consider only topology issues. Actual numbers of link operation cost are not factored in at present. We also assume that, before optimization, each network is fully connected, meaning that each node in one network can find a routing lightpath to reach any other node in that network [19].

To merge the two identical networks, we install some interconnection links so that routing between any two nodes of the two networks can be provided, but with fewer operational fiber links. Operational links refer to the links that will still be in operation after merger. We assume that the interconnections between the two networks only occur at co-located nodes, namely, at the same cities, but not between different cities [20].

B. Problem Formulation

The objective is to derive the minimum number of operational fiber links required while maintaining bi-directional connectivity between any two nodes of the network after merger. We have mentioned that, since the construction cost of the interconnection links varies, the number of interconnection links to be built for consolidation purposes varies, and thus the number of operational fiber links required will also vary. This hinders us from deriving useful information about the minimum requirement on operational fiber links in the merger of two arbitrary identical networks. So, in the following discussions, we will consider the consolidation of two identical optical networks under two extreme cases of different interconnection cost to get some insightful analytical results.

First, we will consider the case in which the cost for the construction of interconnection links is negligible (the full-interconnection case). A simple equation for the minimum number of links required is derived for certain network topologies, whereas for others that do not have an exact solution an upper bound is given. Based on this result, we will then consider the other case in which the interconnection cost is very high such that only two interconnection links will be installed (the two-interconnection case). The optimal location of the two interconnection links will be derived along with the resultant operational fiber links.

III. Full-Interconnection Case

A. Assumptions

Based on the general assumptions in Section II, we further assume that the interconnection links to be installed at the co-located nodes of the two networks cost much less than the operational cost of an original fiber link; thus their cost is negligible. The objective is to derive the minimum number of links required so that there is a path from an arbitrary node to all other nodes in the two networks in which all of the links are directed. Since the cost of installing interconnection links is negligible, we can assume that all co-located nodes are to be interconnected with interconnections. Thus all traffic that goes to and originates from the nodes on the second network will go through the interconnection links and route through the fiber links on the first network, so the original fiber links on the second network can be saved. To find further savings in the number of operational fiber links after the merger, we will first introduce some definitions from graph theory.

B. Definitions and Notation

In graph theory, a bridge is an edge (link) whose removal disconnects a graph [21]. For example, a tree network is a network in which all the links are bridges. A leaf is a vertex (node) of degree 1 [21]. We divide the bridges into two types, namely, TP-I bridges and TP-II bridges. A TP-I bridge is the link associated with a leaf node. The other bridges are TP-II bridges. A cut node (vertex) is a vertex whose removal disconnects the graph [21]. Edge and vertex are common terms in graph theory, whereas link and node are commonly used in routing networks. In this paper, edge and link are interchangeable; so are node and vertex, and graph and network. The following states our notation.

$L_{\min}$—The minimum number of links required after the merger of two identical networks when the interconnection build cost is negligible.

B—The number of bridges in one network before merger.

A_i—The number of cut nodes with the removal of which the graph will be divided into i subgraphs.

C. An Algorithm to Derive $L_{\min}$

The following states an algorithm to derive the minimum number of operational fiber links required after the merger of two identical networks. As the two coexisting networks are identical and all the links in one of the networks have been saved by the installation of the interconnection links, we may concentrate on only one of the two networks.

Step 1: It is obvious that every physical link of bridge type needs two links with opposite directions in order that the two end nodes of the bridge can reach each other. We denote the total number of TP-I bridges and TP-II bridges as B. So $2B$ links in total should be reserved at the places where bridge link occurs. Then remove all the bridges. For the removal of TP-I bridges, we also remove the leaves connected to them at the same time. Then some of the TP-II bridges will become TP-I bridges. Remove them as aforementioned until no TP-I bridge exists. The removal of TP-II bridges is very straightforward. We denote V as the number of vertices that remain after this step, i.e.,

V—The number of nodes remaining after the removal of bridge links as stated in *Step 1*.

Note that V is not equal to the original number of nodes in one network if a TP-I bridge exists. Also the removal of the bridges and leaf nodes are for accounting purposes only; they are still in operation in practice. The bridge type links are identified to require exactly two links of opposite directions after merger.

Step 2: Remove all the cut nodes. Thus the graph is divided into several subgraphs. Then restore the cut nodes to all the subgraphs. The total number of nodes will be larger than V now, as every cut node will be restored back to two or more subgraphs.

Step 3: Check whether all the resultant subgraphs are Hamiltonian or not [21]. A Hamiltonian cycle is a cycle that visits each node exactly once. A graph that contains a Hamiltonian cycle is called a Hamiltonian graph. If all the resultant subgraphs are Hamiltonian, we derive that

$$L_{\min} = 2B + V + \sum_{i=2}^{\infty} A_i(i-1). \quad (1)$$

For one single cut node that divides a graph into i subgraphs, the total number of vertices after *Step 2* is $V+(i-1)$. Since the resultant subgraphs are Hamiltonian, the number of links required is equal to the number of vertices, which is $V + \sum_{i=2}^{\infty} A_i(i-1)$. For the subgraphs that are disconnected by the same cut node, all the traffic between the nodes in them will route through this particular cut node. All the nodes in an arbitrary subgraph can reach and be reached by the cut node since the subgraphs are Hamiltonian. This implies that all the nodes in different subgraphs can reach each other through the cut node.

The removal and restoration of the cut node is a way of decomposition to obtain the Hamiltonian subgraphs. The links forming a Hamiltonian cycle are required to be operational after the merger. The number of links is equal to the number of nodes for each subgraph.

If there are some subgraphs that are not Hamiltonian after *Step 2*, we concentrate on the non-Hamiltonian networks and make the following definitions.

D2 node—A node of degree 2.

Arm—A path consisting of entirely $D2$ nodes and the connecting links plus the two end links connected to the adjacent non-$D2$ nodes.

$j-D2$ arm—An arm with a total of j $D2$ nodes.

For example, in Fig. 1, there is one *3-D2 arm* and two *2-D2 arms* in this non-Hamiltonian graph. We denote N_i as node i and l_{i-j} as the link connecting nodes i and j. One of the two *2-D2 arms* consists of l_{1-2}, N_2, l_{2-3}, N_3, and l_{3-4}, as the dashed line shows.

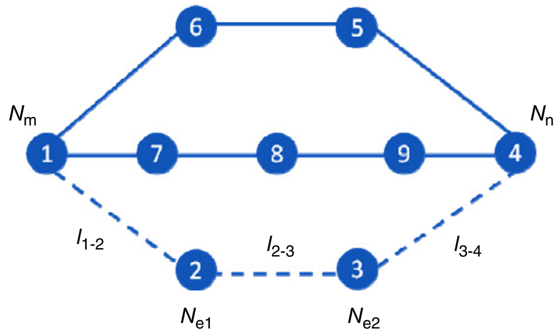


Fig. 1. (Color online) A non-Hamiltonian graph with three arms.

Step 4: Find all the arms first, then remove those arms one at a time until all the subgraphs are Hamiltonian.

Denote M_j as the number of $j-D2$ *arms* deleted, and V_H as the number of remaining vertices in the resultant Hamiltonian graphs. We arrive at the following equation:

$$L_{\min} = 2B + \left[V_H + \sum_{j=1}^{\infty} M_j(j+1)\right] + \sum_{i=2}^{\infty} A_i(i-1). \quad (2)$$

The removal of the arms will be in a strategic way. We will remove one arm first to see if the resultant graph is Hamiltonian. Try another arm if it is not, till all the arms are tested. Then try to remove two arms at a time with different combinations, and so forth. We will prove in the following that the algorithm will provide the minimum required number of fiber links.

Proof. All the $j+1$ links of the $j-D2$ *arm* are required in order that all the $D2$ nodes in the arm can reach other nodes and be reachable from other nodes. So we need at least $V_H + \sum_{j=1}^{\infty} M_j(j+1)$ links. On the other hand, if an arm with end nodes N_{e1} and N_{e2} (they can be the same node) is added between two nodes N_m and N_n of a Hamilton cycle, all the $D2$ nodes in the arm can reach N_m and thus reach all other nodes on the Hamiltonian cycle via the link N_{e1} to N_m. And also, all the $D2$ nodes in the arm can be reached by the other nodes on the Hamiltonian cycle via the link N_n to N_{e2}. To illustrate this, consider the graph in Fig. 1. We view the arm consisting of l_{1-2}, N_2, l_{2-3}, N_3, and l_{3-4} as an attachment to the Hamiltonian cycle consisting of nodes N_1, N_7, N_8, N_9, N_4, N_5, and N_6. Here, N_{e1}, N_{e2}, N_m, and N_n denote N_2, N_3, N_1, and N_4, respectively. Further attachments of arms yield a similar result as long as they are not put onto the previously attached arms. So $V_H + \sum_{j=1}^{\infty} M_j(j+1)$ links are sufficient for all the nodes to be fully connected. Thus, we need exactly $V_H + \sum_{j=1}^{\infty} M_j(j+1)$ links plus the parts denoted by the number of bridges and cut nodes, the first and the third terms in Eq. (2). This completes the proof.

For all networks, including those networks that are still non-Hamiltonian after *Step 4*, it can be proved that the upper bound of $L_{\min}$ is

$$L_{\min} < 2B + 2V + \sum_{i=2}^{\infty} A_i(i-1). \quad (3)$$

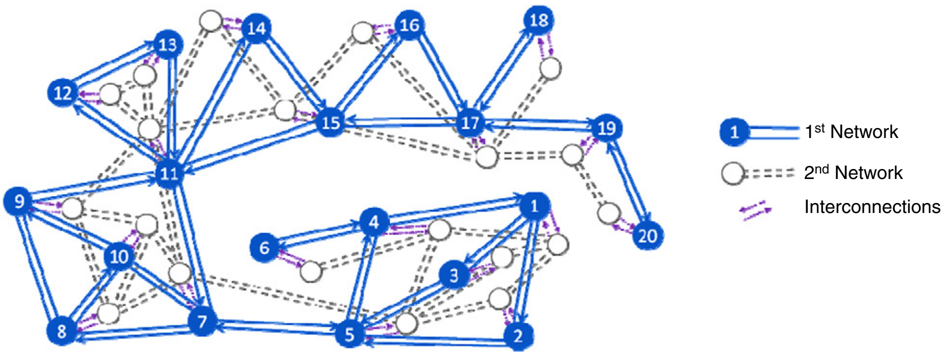


Fig. 2. (Color online) Two coexisting identical networks with interconnections between all the co-located nodes.

The idea in deriving this upper bound is very similar to that in deriving Eq. (2), and thus the derivation is omitted here.

D. Example Illustration

We will illustrate this algorithm using a simple example shown in Fig. 2. We will derive $L_{\min}$ according to the algorithm discussed in this section.

For *Step 1*, we count the number of bridges: $B = 5$. We delete N_6 and $l_{4\text{–}6}$, N_{20} and $l_{19\text{–}20}$, and N_{18} and $l_{17\text{–}18}$, which correspond to the TP-I bridges and the associated leaves. This means that there will be two links of opposite directions at each of $l_{4\text{–}6}$, $l_{19\text{–}20}$, and $l_{17\text{–}18}$. Then we delete also N_{19} and $l_{17\text{–}19}$. And then we delete the only TP-II bridge, $l_{5\text{–}7}$. Four nodes are deleted with the TP-I bridges, so $V = 20 - 4 = 16$. After *Step 2*, we derive five subgraphs, namely, the subgraphs consisting of nodes 1–2–3–4–5, 7–8–9–10–11, 11–12–13, 11–14–15, and 15–16–17. There are two cut nodes: N_{15} with two subgraphs ($A_2 = 1$), and N_{11} with three subgraphs ($A_3 = 1$). All of the subgraphs are Hamiltonian except the first one. We continue to *Step 4*, and delete the arm consisting of N_2, $l_{1\text{–}2}$, and $l_{2\text{–}5}$. Thus all the subgraphs are Hamiltonian. $M_1 = 1$. $V_H = 15$. So the minimum number of links required is

$$L_{\min} = 2B + \left[V_H + M_j(j+1)\right] + \sum_{i=2}^{\infty} A_i(i-1) = 30.$$

Thus the number of links is reduced by 72.2% ($= \frac{108-30}{108}$) for the two specific networks. One feasible result is illustrated in Fig. 2 as indicated by the 30 links with arrows.

In this section, we investigated the minimum number of links required in consolidating two overlapped networks to make every two nodes bi-directionally connected under the assumption that all the co-located nodes are interconnected. We proposed an algorithm to derive the minimum number of links and found that it is at least the number of the nodes or at least twice the number of the bridges in one of the networks. We now move on to investigate the case in which only two interconnection links are used, assuming that the interconnection construction cost is very high.

IV. Two-Interconnection Case

In this section, we will discuss the merger of two optical networks with only two interconnections based on the results from Section III.

A. Assumptions

When the installation of interconnection links is assumed to be of very high cost, we tend to employ as few interconnections as possible. We have assumed that all the links including the original operational fiber links and the newly added interconnections are all directed; therefore a minimum of two interconnection links are required for the consolidation [22,23]. The two interconnection links need to be of reverse directions so that traffic can route from one network to the other.

In Section III, we derived the minimum number of fiber links required ($L_{\min}$) in the merger of two networks when all the co-located nodes are installed with interconnection links. Optimal solutions of the remaining operational fiber links with traffic routing directions can also be obtained. All the resultant $L_{\min}$ fiber links are in one network, whereas all the links in the second network are suspended as all traffic flows that originate from or are destined for the nodes on this network will route through the interconnection links. When the number of interconnection links is reduced to two, which is the minimum requirement, more fiber links are needed to provide bi-directional connection between any two nodes of the two networks. One simple and straightforward way is to keep the optimal $L_{\min}$ fiber links of the first network as derived in Section III and also retain the same links in the second network but with reverse directions. Then, after the two interconnection links of reverse directions (one routes the traffic from network A to network B and the other from network B to A) are installed, traffic between any two nodes in the two networks can be supported. Here, we need $L_{\min}$ operational fiber links for each network, and thus $2L_{\min}$ in total, plus the two newly installed interconnection links. However, this is not an optimal solution. Based on the optimal solution for one network derived using the algorithm proposed in Section III, we then assume that the other network shares the same remaining links but with reverse directions. We will analyze the two interconnection locations in order to achieve maximum savings in the number of fiber links for the merger of two optical networks.

B. Analysis on the Optimal Location of the Two Interconnection Links

We have assumed that the two mirror networks are first optimized to be with the same remaining $L_{\min}$ operational links but of reverse directions for all corresponding mirror links. The following propositions and corollaries to be derived are all based on this assumption. With two interconnection links properly installed, a further saving in the number of fiber links is possible, i.e., fewer than $2L_{\min}$ operational links are needed to provide bi-directional connection for any two nodes in the two networks. We are going to discuss the optimal location for the two interconnection links to provide maximum savings in the number of operational fiber links for consolidation when only two interconnections are to be installed.

We denote these two networks as network A and network B and define some notation as follows.

N_i^A, N_i^B—Two co-located nodes i of network A and network B, respectively.

$l^A(i,j)$, $l^B(j,i)$—The link going from node i to node j in network A and the corresponding mirror link going from node j to node i in network B.

l_i^{AB}—The interconnection link installed at node i that goes from network A to network B.

Note that all the propositions and corollaries that we are going to derive are based on the following assumption.

Assumption. Use the algorithm in Section III for the two mirror networks and assume the two networks are optimized to attain the same $L_{\min}$ operational links but of reverse directions for all corresponding mirror links. Suppose that only two interconnection links are to be installed.

Proposition 1. *The two interconnection links should not be installed at the same location.*

Proof. Refer to Fig. 3. Suppose that the two interconnection links are installed between N_m^A and N_m^B, and that there is a link $l^A(i,j)$ in network A that can be further suspended. So there must be a path that can route the traffic from N_i^A to N_j^B through the two interconnection links rather than $l^A(i,j)$:

$$N_i^A \to (\ldots) \to N_m^A \xrightarrow{l_m^{AB}} N_m^B \xrightarrow{l_m^{BA}} N_m^A \to (\ldots) \to N_j^A.$$

This implies that there is a path in network A that supports the traffic from N_i^A to N_j^B:

$$N_i^A \to (\ldots) \to N_m^A \to (\ldots) \to N_j^A.$$

So $l^A(i,j)$ can be suspended in the prior optimization of network A, which conflicts with the fact that network A is already optimized with the minimum number of fiber links. Thus with the two interconnection links installed at the same location, no further saving in the number of fiber links is possible, and the required number of fiber links is $2L_{\min}$.

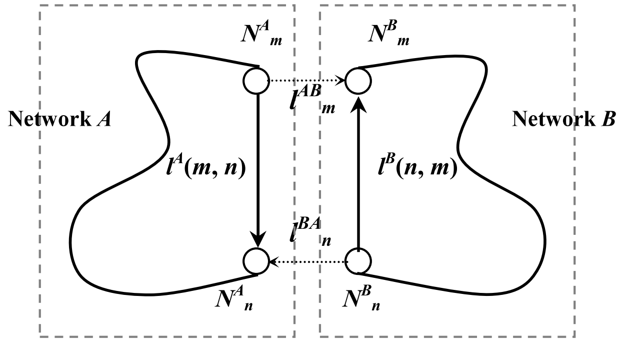


Fig. 3. Two interconnection links installed at neighboring nodes m and n.

On the other hand, suppose that we install the two interconnection links at two neighboring nodes m and n as shown in Fig. 3. Then we claim that $l^A(m,n)$ and $l^B(n,m)$ can be further suspended, which results in saving two more links. $l^A(m,n)$ is the only path in network A for N_m^A to access N_n^A because, if there is another path that can provide routing for the two nodes, $l^A(m,n)$ should have been suspended in the optimization of network A, as proved above. But with the two interconnection links installed, the traffic from N_m^A to N_n^A can be routed through the path

$$N_m^A \xrightarrow{l_m^{AB}} N_m^B \to (\ldots) \to N_n^B \xrightarrow{l_n^{BA}} N_n^A.$$

So $l^A(m,n)$ can now be suspended, as the traffic that originally goes through $l^A(m,n)$ can be routed in the above path instead. A similar situation happens with $l^B(n,m)$. Therefore, a saving of at least two more links can be achieved with the interconnection links installed at two neighboring nodes. This proves Proposition 1. The following corollary follows.

Corollary 1. *If the two interconnection links are installed at two neighboring nodes, a saving of exactly two more links can be achieved, resulting in* $2(L_{\min}-1)$ *total links.*

Proposition 2. *If there are cut nodes in the two optimized networks, the two interconnection links should not be installed at any cut node in order to save more operational fiber links.*

Proof. Suppose that one of the interconnection links is installed at a cut node p and that the other is installed at any other arbitrary node. Suppose that the removal of node p disjoins network A into two subnetworks, network A_1 and A_2, respectively. It is fair to assume that the other interconnection link is installed at network A_2. If node p disjoins the network into more subnetworks, just regard the one with the second interconnection as A_2, and the rest as A_1. Then possible fiber link suspension can only occur in network A_2 (as well as network B_2). No link suspension is possible in network A_1. This can be easily proved by using Proposition 1. For network A_1, it can be regarded that the two interconnection links are both installed at node p.

Since node p is a cut node, there must be at least one adjacent node q in A_1 that connects node p with $l^{A1}(q,p)$. If we move the interconnection link from the cut node p to node q, which is one hop from p, then, for network A_1, it can be regarded that the two interconnection links are installed at

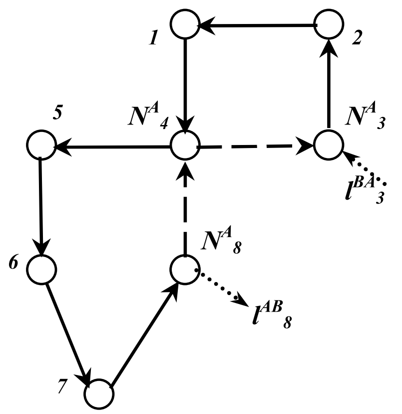


Fig. 4. A dual-ring topology.

two neighboring nodes. This will result in a saving of one link ($l^A(q,p)$) for network A_1, and the suspended links in network A_2 will not be affected; see Corollary 1. This proves Proposition 2, and we have the following corollary.

Corollary 2. *If the two interconnection links are installed at the two sides (network A_1 and network A_2) of a cut node, and are both one hop from the cut node, it will result in a saving of two links for network A . This results in $2(L_{\min}-2)$ total links for the two networks after consolidation.*

For example, Fig. 4 shows a dual-ring topology which contains one cut node N^A_4. Network A is already optimized with the minimum number of fiber links. Suppose that network B is identically optimized but with reverse link direction. With interconnection links installed between N^A_8 and $N^B_8(l^{AB}_8)$, and N^B_3 and $N^B_3(l^{BA}_3)$, further suspension of $l^A(4,3), l^A(8,4)$ and $l^B(3,4)$, $l^B(4,8)$ is possible. This is one of the optimal solutions for the interconnection locations.

From Corollary 2, it follows that, when there is a chain of n directly connected cut nodes which forms a bus topology, if we install the two interconnection links to the nodes which are both one hop from the two most apart cut nodes, the savings in the number of fiber links would be $2(n+1)$ for the two networks. $2(n-1)$ of them are between the directly connected cut nodes. With this result, we come to Proposition 3.

Proposition 3. *If there are cut nodes in the two optimized networks, the two interconnection links should be installed at the nodes which are one hop from certain cut nodes.*

This is a necessary condition for the merger of the two optimized networks in order to achieve maximum savings in the number of operational fiber links. We will prove this by considering the following two cases. Note that we have excluded the possible cases that put the two interconnection links at the same node or at any cut node in Propositions 1 and 2, respectively. From Corollaries 1 and 2, we can also exclude the case that puts the two interconnection links at two neighboring nodes.

Case I. Suppose that neither of the two interconnection links is installed at one hop from any cut node. Then find a path that contains groups of directly connected cut nodes in bus topology between the two interconnection nodes, and maximizes $\sum n_k$. n_k is the node number of the k^{th} group of directly connected cut nodes. The path cannot go through a node twice. The savings in the number of fiber links would be $\sum 2(n_k-1)$ for the two networks.

But if we move the two interconnection links to the two nodes where both are one hop from the most apart cut nodes in the path, a saving of two more links in network A can be achieved. If there are only isolated cut nodes in between the two interconnection locations, installing the two interconnections as described in Corollary 2 leads to more savings in the number of fiber links. Here, the two interconnection links are both one hop from a cut node. So in order to achieve maximum savings in the number of fiber links for consolidation, it is not an optimal solution if neither of the two interconnection links is installed at one hop from a cut node.

Case II. Suppose that one interconnection link is installed at one hop away from a cut node, and that the other is installed at any arbitrary node but not one hop from any cut node. Similar to *Case I*, find a path that contains groups of directly connected cut nodes in bus topology between the two interconnection nodes, and maximizes $\sum n_k$. Now the first interconnection link is already installed at one hop away from one end node of the path.

If we move the second interconnection link to the node which is one hop away from the other end node of the path, a saving of one more link in network A can be achieved. The two interconnection links are both installed at one hop from a cut node.

This proves Proposition 3, namely, that the two interconnection links should be installed at nodes which are one hop from certain cut nodes.

With Proposition 3, we come to the final conclusion for the location of two interconnection links for the merger of two identical optimized networks. Find a path that maximizes $L_s = \sum 2(n_k-1)$, and install the two interconnection links at the two nodes that are both one hop from the most apart cut nodes in the path. $(L_s+2\times 2)$ is the maximum further saving in the number of fiber links that can be achieved. One hypothetical topology is illustrated in Fig. 5 as an example. The shaded nodes are cut nodes, and there are four groups of directly connected cut nodes. The path that contains $G1$ and $G3$ would be chosen and $(L_s+2\times 2) = \sum 2(n_k-1)+4 = 10$ links can be further saved with the two interconnection links installed as l^{AB}_3 and l^{BA}_{16}. The five saved links of network A are denoted as dashed lines. Note that moving l^{BA}_{16} to l^{BA}_{18} would be another optimal solution.

We then examine the consolidation of the NSFNET network topology as illustrated in Fig. 6. We first merge the two networks using the algorithm derived in Section III. Then, by applying Corollary 1, we find that installing two interconnection links at some particular neighboring nodes yields an optimal solution for the merger to achieve maximum savings in the number of fiber links. A feasible solution is shown in Fig. 7. With the two interconnection links installed, a Hamiltonian cycle that covers all the nodes of the two networks is formed. Extensive simulation work has also been carried out on various network topologies to test the validity of the theoretical findings presented in this paper [20].

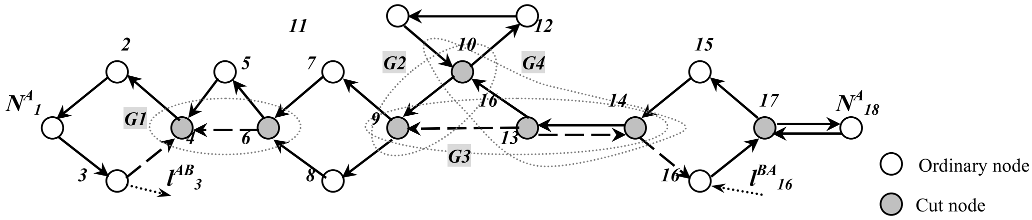


Fig. 5. A hypothetical topology with four groups of directly connected cut nodes.

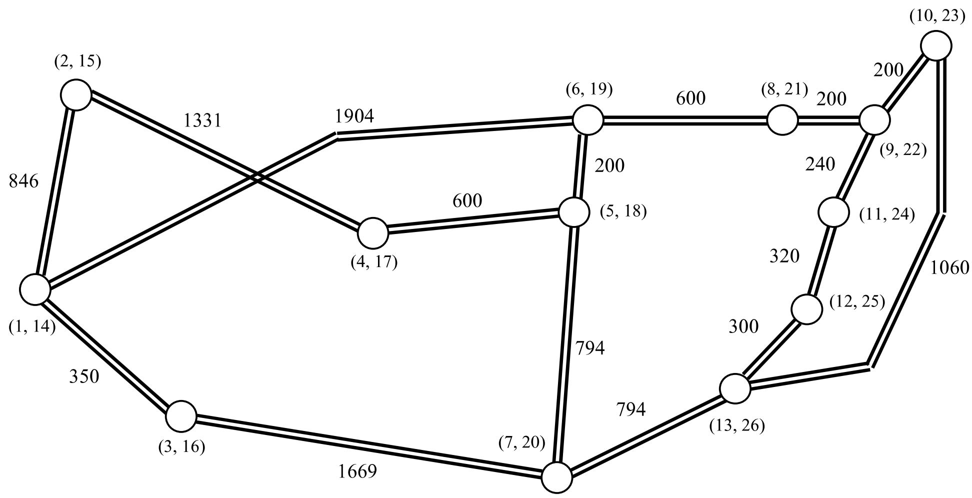


Fig. 6. Two NSFNET networks. Node (a, b) is node a in network A and node b in network B.

In this section, we investigated the merger of two optical networks under the assumption that the interconnection construction is very costly such that only two interconnections are installed. We discussed the installation locations of the two interconnection links to achieve maximum savings in the number of operational fiber links based on the results from Section III. We proved that the two interconnection links should not be installed at the same node for any network. For networks that have cut nodes, the interconnection links should not be installed at cut nodes. We conclude that the path that contains the maximum number of directly connected cut nodes in different groupings should be found, and installing the two interconnection links to one hop from the two most apart cut nodes will be an optimal solution for the merger of two networks.

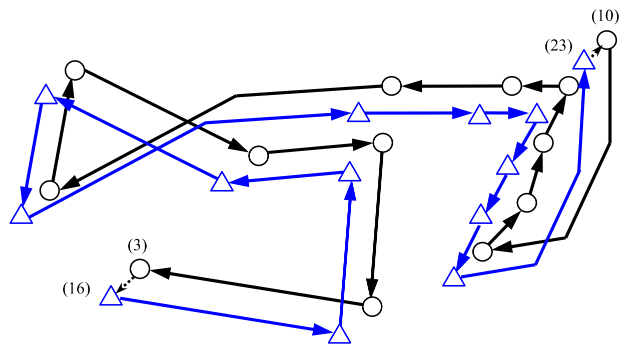
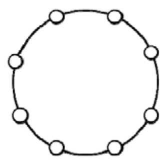
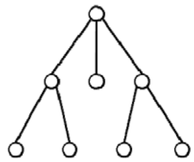
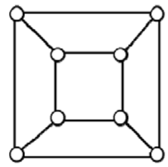
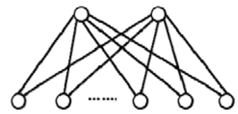


Fig. 7. (Color online) Two NSFNET networks after merger. Interconnection links installed at node (3, 6) and node (10, 23).

V. Connectivity Efficiency of Network Topology

In this section, we consider the connectivity efficiency of different network topologies. We will generalize the requirements on the number of operational fiber links over the number of nodes of an arbitrary network based on previous discussions. This ratio is defined as the connectivity efficiency of a network topology. It varies with the topology of the networks when the number of interconnection links is fixed. A directed network is connected if there is a directed path from node i to node j and a directed path from node j to node i for an arbitrary pair of nodes i and j. Whatever the topology of the original network is, the merged network is directed and connected. However, the number of fiber links retained after merger is different for networks with the same number of nodes but different topologies. The ratio gives the topological efficiency in connecting the network nodes to provide any node to any node connectivity. The lower the ratio, the more efficient is the network topology. For the merger of two identical networks with N nodes each, we define L as the minimum number of operational links required to make the merged network connected. We have mentioned that, for the merger

TABLE I
CONNECTIVITY EFFICIENCY RATIO OF DIFFERENT NETWORK TOPOLOGIES

Network Topology / Connectivity Efficiency L/N		Ring 	Tree 	Hamiltonian 	$K_{2,n}$ 
No. of Inter-Connections	Full	1	2	1	2
	Two	2	[2, 4)	2	4

of two networks with a specific topology, when the number of interconnections to be installed varies, L also varies; thus L/N varies. On the other hand, if the number of interconnection links is fixed, it is interesting to find that L/N also varies quite significantly for the merger of networks of different topologies. This is from the topological point of view. We will discuss the value of L/N, the topology efficiency ratio, for the two cases of full interconnection in Section III and also the case of only two interconnections in Section IV, respectively. We will find the range of L/N for these two fixed-interconnection cost cases.

A. Full-Interconnection Case

For the full-interconnection case, $L = L_{\min}$. We will first derive the lower bound of L/N. From Eqs. (1)–(3) of Section III, we can find that the lower bound of L/N is only possible to achieve when $L_{\min} = 2B + V + \sum_{i=2}^{\infty} A_i(i-1)$. If there are no bridges or cut nodes in the network, we directly come to *Step 3* in Section III. And if the graph (network) is Hamiltonian, then $L_{\min} = V = N$. Thus $L/N = 1$. This is the lower bound of L/N. In other words, we can have the smallest value of L/N if the two networks to be consolidated are Hamiltonian graphs, which have a cycle that visits every node exactly once and returns to the starting node. Thus, in the previous consolidation, it can be found that the optimization can be rendered to finding Hamiltonian cycles, if they exist, in the overlapped area.

For the upper bound of L/N, from Eq. (3) we know that $L = L_{\min} < 2N$. So the range of L/N, the number of operational fiber links required after the merger over the number of nodes of a network, is [1, 2). A tree network is an example where L/N tends to the upper bound of 2. For a tree network with N nodes, there are $N-1$ bridges. The merger of two identical tree networks will result in $L = 2(N-1)$ links from the algorithm in Section III. So $L/N = 2-(2/N)$ which tends to 2 as the number of nodes N increases.

B. Two-Interconnection Case

Similarly, we derived that, for the two-interconnection case in Section IV, the range of L/N, the number of operational fiber links and interconnections required after the merger over the number of nodes of a network, is [2, 4). When the two networks to be merged have Hamiltonian paths, the lower bound $L/N = 2$ can be achieved. For the upper bound, an example is a complete bipartite graph $K_{2,n}$ which has $N = (2+n)$ nodes and $2n$ links [18]. All the n nodes are $D2$ nodes, and there are *n1-D2 arms*. From Section III, $L_{\min} = 2n$ for the full-interconnection case. When the number of interconnection links is reduced to two, installing these two interconnection links at two neighboring nodes is an optimal solution because there are no cut nodes in this network. Thus we need $(2n-1)$ links for either network when there are only two interconnection links. So the total number of operational fiber links and interconnections is $L = 4n$. When n is very large, L/N tends to the upper bound of 4.

Table I shows some of the network topologies we have discussed and lists the values of what their connectivity efficiency ratio approaches when the number of network nodes is very large. The Hamiltonian networks give the best connectivity efficiency. A ring is the simplest Hamiltonian network.

VI. PROTECTION OF THE MERGED NETWORK

In the previous discussions, we concentrated on the minimum requirements of operational fiber links to provide connectivity for the nodes in the two networks under two circumstances of different interconnection costs. To take into account more practical considerations, we will discuss the protection issues of the merged network against link failure in this section. Service providers have to use backup or redundant network resources to protect their network against the risk of link failure. But at the same time they need to reduce their investment by using a minimum number of redundant resources [24–26].

Single fiber cut is the most common failure scenario in the operation of fiber optics networks [27]. We will provide single link fault protection schemes for the two cases discussed in Sections III and IV.

A. Full-Interconnection Case

For the full-interconnection case, suppose that the merged network is in operation with the minimum $L_{\min}$ fiber links. All the $L_{\min}$ links are in one network and all the co-located nodes are installed with two interconnection links with reverse

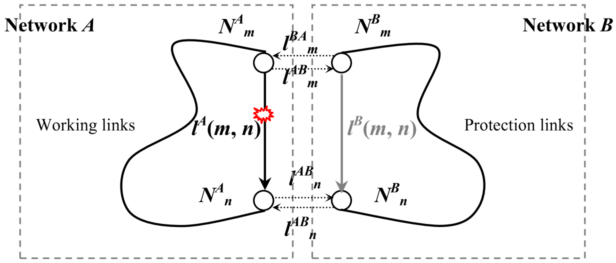


Fig. 8. (Color online) Protection scheme for the full-interconnection case when the interconnection build cost is negligible.

directions. We can provide protection for any single link failure of the $L_{\min}$ fiber links by turning on the corresponding $L_{\min}$ links in the second network. This is sufficient and necessary. This provides 1:1 protection for the merged network [28]. It is obvious that we have enough fiber links available in the second network for protection purposes; thus it is a sufficient condition. We will prove in the following that all the $L_{\min}$ protection links need to be used.

Proof. Assume that one of the $L_{\min}$ links for the single-failure protection case above can be removed and that only $(L_{\min}-1)$ protection links are needed. The notation in Fig. 8 is the same as in Section IV. Suppose that an arbitrary protection link $l^B(m,n)$ can be removed. Then traffic from N_m^A to N_n^A and from N_m^B to N_n^B cannot be supported if the single link failure happens at $l^A(m,n)$. If there is still a path that can provide a connection for traffic from N_m^A to N_n^A, $l^A(m,n)$ should have been suspended in the prior optimization of network A, as discussed in the proof of Proposition 1 in Section IV.

This proves the claim that all the $L_{\min}$ protection links are necessary for the 1:1 protection scheme.

For the merged network with minimum $L_{\min}$ links, the optimal solution for 1:1 protection can be different from our proposed scheme, but the number of protection links required is always $L_{\min}$.

B. Two-Interconnection Case

For the two-interconnection case in Section IV, the merged network consists of three parts, namely, the two interconnection links, the remaining fiber links in network A, and the links in network B. The fiber links in network A and B are located at the same places but with reverse directions. To protect against single fiber cut of this merged network, we can turn on a set of protection links in network A and B and make sure that all the protection links in network A (network B) are identical to the working links of network B (network A). Also, two more interconnection links should be installed at the same locations as the working interconnections but with reverse directions. This provides a simple single-failure protection scheme for the merged network, and there is no redundant protection link in this scheme. The proof is quite straightforward, and thus is omitted here. But the problem is that there may not be enough fiber links available to be turned on for protection purposes as the merged network is already using a considerably large number of working links. For these situations, we will claim that no protection scheme for single failure of an arbitrary fiber link is available. For example, the original two identical tree networks with 8 nodes and 14 links for each network in Fig. 9(a) are optimized to be the network in Fig. 9(b) with two interconnection links (the dashed links). The optimized network operates with 20 fiber links, whereas 28.6% (=8/28) of the fiber links are saved. We are not able to provide an arbitrary single link failure protection scheme for this network, since there are no more fiber links available at some places where fiber link failure happens. For example, if the single link failure happens at any double lines (any one of the two reversely directed links) of Fig. 9(b), no backup protection link is available. But we can still protect against single link failure at any single line in Fig. 9(b). In that case, we may install more interconnection links for protection purposes.

VII. Conclusions

In this paper, we have investigated the consolidation of two optical networks with similar topology and geographical

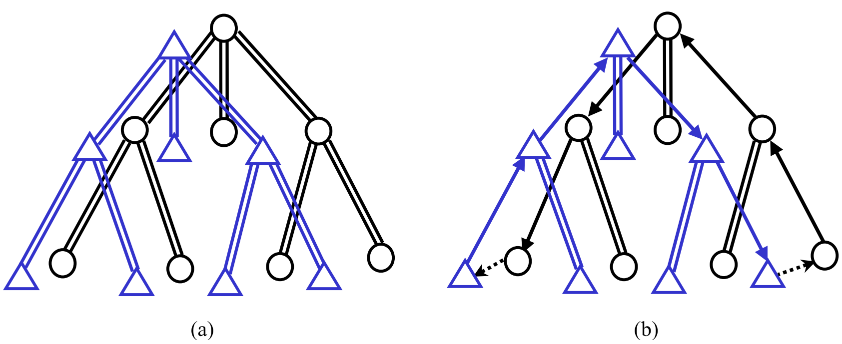


Fig. 9. (Color online) The consolidation of two identical tree networks with two interconnection links. (a) Two identical networks: network A in blue color and network B in black; double line means two operational fiber links with reverse direction. (b) The merged network with two interconnection links installed at two most apart cut nodes.

coverage. Interconnection links are installed at strategic locations such that some redundant links in the overlapping areas can be suspended. We considered the merger in two scenarios. First, we assumed that the construction of interconnection links is of very low cost such that all the co-located nodes are installed with interconnections. We developed an algorithm to derive the minimum number of operational fiber links required to provide bi-directional connections between any two nodes of the merged network. Based on this result, we then moved on to consider the case with only two interconnection links, which is the minimum required number of interconnections. The optimal locations of the two interconnection links with corresponding resultant operational fiber links were given. We discussed the connectivity efficiency concerning different network topologies. We showed that Hamiltonian networks provide the best efficiency, and thus, in the previous consolidation, the optimization can be rendered to finding a Hamiltonian cycle, if it exists, in the overlapped area.

We have focused more on the minimum requirement of operational fiber links to provide connectivity for any two nodes of the consolidated network. For more practical considerations, the protection of optical networks is also of great importance. We discussed the protection schemes for the most common failure scenario of single fiber cut in Section VI. We claimed that a further L protection links (the minimum number of operational links required to provide bi-directional traffic between any two nodes) are required for protecting against an arbitrary fiber link failure. The improvement of protection schemes will be part of our future work.

Acknowledgments

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