Optimal Location Analysis of Two Interconnections for the Consolidation of Two Networks

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Abstract: In the merging of two identical optimized optical networks with two minimal interconnection links, we analyze the optimal locations of the two interconnection links to achieve maximum saving in fiber links.

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1. Introduction

Telecommunication infrastructure has been growing extremely fast in the last decade. Substantial amount of fiber optics networks have been constructed by different carriers. The network infrastructures may overlap extensively in one region as carriers want to compete with each other in more profitable areas. After the telecommunications deregulation, merge and acquisition of competitive operators are often and lead to the consolidation of two networks. It is also possible that more fiber links can be saved when transmission links at higher data rate (e.g. 40Gb/s or 100Gb/s link) [1] are employed to achieve high utilization of network resources. In the consolidation of two networks, through traffic grooming and rerouting, some of the links can be suspended. Though the deployed fibers of the suspended links can not be reallocated, the operation cost of the regenerator site can be saved [2]. Also, inter-connectivity between the two networks can be supported. We have proposed an algorithm to derive the minimum number of fiber links required in the merging of two coexisting networks with interconnection links installed at all co-located nodes [3]. This is the maximum saving that can be achieved, assuming that any two nodes in the two networks are bi-directionally connected. On the other hand, it is obvious that a minimum of two interconnection links are required for the consolidation. The two interconnection links need to be of reverse directions so that traffic can route from one network to the other. We investigate the locations of the two interconnection links to provide maximum saving in fiber links for the merging of two identical optical networks. When there are articulation nodes, with the removal of which the network will be divided into two or more sub-networks [4], we show in this paper that the two interconnection links should be installed at the nodes which are one hop from two certain articulation nodes.

2. Problem formulation

We model the existing optical networks as directed planar graphs as in [3]. It is assumed that there are two identical optical networks that their nodes and fiber links are all colocated, and the cost of every fiber link is identical. When all the co-located nodes are installed with interconnection links, an optimization algorithm was developed. With the minimum number of fiber links required, L_{min} derived, optimal solutions of the remaining operational fiber links with traffic routing directions can also be obtained. All the resultant L_{min} fiber links are in one network, while all the links in the other network are suspended as all traffic originated from or destinated to the nodes on this network will route through the interconnection links. The consolidation optimization is reduced to a single-network optimization to achieve a two-link connected network with minimum number of links. Whereas when the number of interconnection links is reduced to two, which is the minimal requirement, more fiber links are needed to provide bidirectional connection between any two nodes of the two networks. If we keep the optimal L_{min} fiber links of the first network and also the same links in the second network but with reverse directions, traffic between any two nodes in the two networks can be supported with two interconnection links installed as long as they are of reverse directions. But this is not the optimal solution. Based on the optimal solution for one network derived using our proposed algorithm [3], we then assume the other network shares the same remaining links but with reverse directions, we will analyze the two interconnection locations in order to achieve maximum saving in fiber links for the merging of two optical networks.

3. Location analysis of the two interconnection links

We will derive optimal locations for the two interconnection links so that maximum saving in fiber links can be achieved in the merging of two optimized networks. Assume that the two mirror networks are first optimized to be with the same remaining operational links but of reverse direction for all corresponding mirror links. With two interconnection links properly installed, further saving in fiber links is possible. We regard these two networks as network A and network B and define some notations as follows:

$\boldsymbol{N^A_i, N^B_j}$ – two co-located nodes i of network A and network B respectively.

$\boldsymbol{l^A(i, j), l^B(j, i)}$ – the link going from node i to node j in network A and the corresponding mirror link going from node j to node i in network B.

$\boldsymbol{l^{AB}_i}$ – interconnection link installed at node i and goes from network A to network B.

Theorem 1. The two interconnection links shall not be installed at the same location.

Proof: Suppose the two interconnection links are installed between N^A_m and N^B_m, and there is a link $l^A(i, j)$ in network A that can be further suspended. So there must be a path that can route the traffic from N^A_i to N^B_j through the two interconnection links rather than $l^A(i, j)$:

$$N^A_i \to (...) \to N^A_m \xrightarrow{l^{AB}m} N^B_m \xrightarrow{l^{BA}m} N^A_m \to (...) \to N^A_j$$

Which implies that there is a path in network A that supports the traffic from N^A_i to N^B_j:

$$N^A_i \to (...) \to N^A_m \to (...) \to N^A_j$$

So $l^A(i, j)$ can be suspended in the prior optimization of network A, which conflicts with the fact that network A is already optimized with minimum fiber links. Thus with the two interconnection links installed at the same location, no further saving in fiber links is possible, the required number of fiber links is 2 L_{min}.

On the other hand, suppose we install the two interconnection links at two neighboring nodes m and n as shown in *Fig. 1*. Then we claim that $l^A(m, n)$ and $l^B(n, m)$ can be further suspended which results in two more links saving. $l^A(m, n)$ is the only path in network A for N^A_m to access N^A_n as if there is another path that can provide routing for the two nodes, $l^A(m, n)$ should have been suspended in the optimization of network A as proved above. But with the two interconnection links installed, the traffic from N^A_m to N^A_n can be routed through the path: $N^A_m \xrightarrow{l^{AB}m} N^B_m \to (...) \to N^B_n \xrightarrow{l^{BA}n} N^A_n$. So $l^A(m, n)$ can now be suspended as the traffic that originally goes through $l^A(m, n)$ can be routed in the above path instead. Similar situation happens to $l^B(n, m)$. Therefore, two more links saving can at least be achieved with the interconnection links installed at two neighboring nodes. This proves *Theorem 1*. Actually, we have already proved the following corollary.

Corollary 2. If the two interconnection links are installed at two neighboring nodes, exactly two more links saving can be achieved, resulting in $2(L_{min} - 1)$ total links.

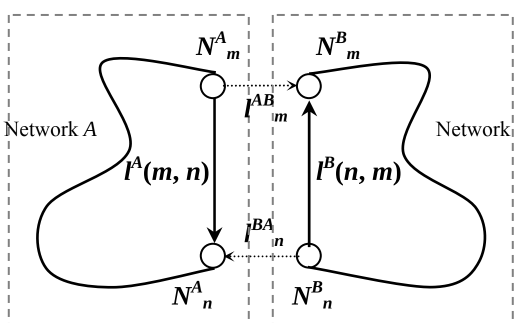

Fig. 1. Two interconnection links installed at neighboring nodes m and n.

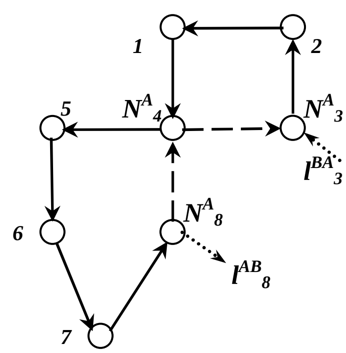

Fig. 2. A dual-ring topology.

Theorem 3. If there are articulation nodes in the two optimized networks, the two interconnection links should be installed at the nodes which are one hop from certain articulation nodes.

This is a necessary condition. We will prove this by considering the following two cases.

Case i. Suppose one of the interconnection links is installed at an articulation node p and the other is installed at any other arbitrary node. Suppose the removal of node p disjoins network A into two sub-networks, network A_1 and A_2 respectively. It is fair to assume that the other interconnection link is installed at network A_2. If node p disjoins the network into more sub-networks, just regard the one with interconnection as A_2, and the rest A_1. Then possible fiber link saving can only occur in network A_2 (as well as network B_2). No link saving is possible in network A_1. This can be easily proved by using *Theorem 1*. Because for network A_1, it can be regarded as the two interconnection links are both installed at node p.

As node p is an articulation node, there must be at least one adjacent node q in A_1 that connects to node p with $l^{A_1}(q, p)$. If we move the interconnection link from the articulation node p to node q, which is one hop from p, then for network A_1, it can be regarded as the two interconnection links are installed at two neighboring nodes. This will result in one link saving of $l^A(q, p)$ for network A_1 and the saving in network A_2 will not be affected as from *Corollary 2*. With *Case i*, we have the following two corollaries.

Corollary 4. The interconnection links shall not be installed at articulation nodes.

Corollary 5. If the two interconnection links are installed at the two sides (network A_1 and network A_2) of an articulation node, which are both one hop from the articulation node, it will result in two links saving for network *A*. Thus results in $2(L_{min} - 2)$ total links for the two networks.

For example, *Fig. 2* shows a dual-ring topology which contains one articulation node N^A_4. Network *A* is already optimized with minimum fiber links. Suppose Network *B* is identically optimized but with reverse link direction. With interconnection links installed between N^A_8 and N^B_8 (l^{AB}_8), N^B_3 and N^B_3 (l^{BA}_3), further saving of $l^A(4, 3)$, $l^A(8, 4)$ and $l^B(3, 4)$, $l^B(4, 8)$ can be obtained. This is actually one of the optimal solutions of the interconnection locations.

From *Corollary 5*, it follows that when there are *n* directly connected articulation nodes which forms a bus topology, if we install the two interconnection links to the nodes which are both one hop from the two most apart articulation nodes, the saving in fiber links would be $2(n + 1)$ for the two networks. $2(n - 1)$ of them are between the directly connected articulation nodes.

Case ii. Suppose neither of the two interconnection links is installed at one hop from any articulation nodes. Note that they should not be installed at neighboring nodes or any articulation nodes either. Then find a path that contains groups of directly connected articulation nodes in bus topology between the two interconnection nodes, and maximizes $\sum n_k$. n_k is the node number of the k^{th} group of directly connected articulation nodes. The path can not experience a node twice. The saving in fiber links would be $\sum 2(n_k - 1)$ for the two networks. But if we move the two interconnection links to the two nodes where are both one hop from the most apart articulation nodes in the path, two more link saving in network *A* can be achieved. If there are only isolated articulation nodes in between the two interconnection locations, install the two interconnections as described in *Corollary 5* improves fiber saving. This proves *Theorem 3*.

With *Theorem 3*, we come to the final conclusion for the location of two interconnection links for the merging of two identical optimized networks. Find a path that maximizes $L_s = \sum 2(n_k - 1)$, and install the two interconnection links at the two nodes where are both one hop from the most apart articulation nodes in the path. $(L_s + 2\times2)$ is the maximum further saving in fiber links that can be achieved. One hypothetic topology is illustrated in *Fig. 3* as an example. The shaded nodes are articulation nodes, and there are four groups of directly connected articulation nodes. The path contains *G1* and *G3* would be chosen and $(L_s + 2 \times 2) = \sum 2(n_i - 1) + 4 = 10$ links can be further saved with the two interconnection links installed as l^{AB}_3 and l^{BA}_{16}. The five saved links of network *A* are denoted as dashed lines. Note that move l^{BA}_{16} to l^{BA}_{18} would be another optimal solution.

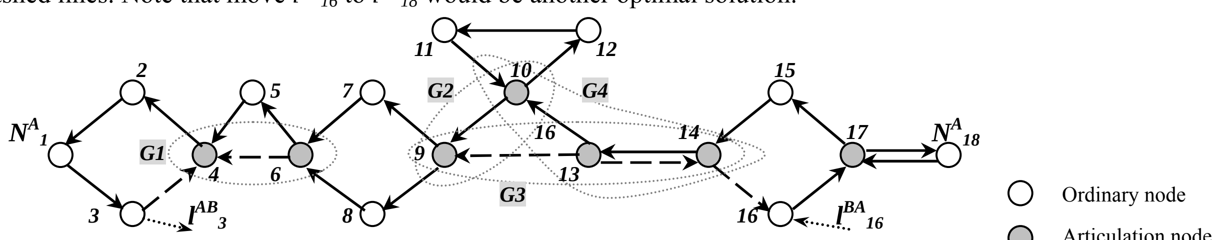

Fig. 3. A hypothetic topology with four groups of directly connected articulation nodes.

4. Conclusions

We investigated the locations of two interconnection links for the merging of two optimized fiber optic networks. We proved that the two interconnection links shall not be located at the same node for any networks. For networks that have articulation nodes, the interconnection links shall not be installed at articulation nodes. We conclude that the path contains maximum number of directly connected articulation nodes in different groupings should be found, and install the two interconnection links to one hop from the two most apart articulation nodes will be an optimal solution for the merging of two networks.

5. References

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