Analysis and Measurement of Root-Mean-Squared Bandwidth of Cross-Phase-Modulation-Induced Spectral Broadening

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***Abstract*— The root-mean-squared bandwidth of spectral broadening induced by cross-phase modulation is derived analytically for a two-channel system consisting of a pump and probe channel. The pump channel is randomly modulated by binary data and the probe channel is a weak continuous-wave channel. The analytical expression is verified by experiments.**

***Index Terms*— Cross-phase modulation, fiber nonlinearities, WDM.**

I. Introduction

WAVELENGTH-DIVISION-MULTIPLEXING (WDM) techniques are revolutionizing the broadband networks by offering enormous communication bandwidth. But the WDM systems also give rise to new undesirable nonlinear interchannel effects such as four-wave mixing (FWM) and cross-phase modulation (XPM) [1] when multiple channels are transmitted through the same optical fiber. Affecting systems in dispersive fiber, XPM is an effect in which the optical phase of each channel is modulated by the overall optical power in the fiber because of its power-dependence of refractive index. Consequently, the optical spectrum of the WDM channel is broadened [2], limiting the transmitting distance of an optical signal in dispersive fiber. Properties of XPM induced spectral broadening can be retrieved from a simple setup using only one modulated pump channel (or signal channel) and a continuous-wave (CW) probe channel [3]–[7]. As summarized in [2], previous studies of XPM induced spectral broadening focus on single or periodic pulse interaction but a WDM system always transmits a random data stream. Previous work [8] for random data stream is a simple qualitative study about frequency modulated laser with residual intensity modulation. Modulated by the random data of the pump channel, the phase of the probe is a random process and its statistical properties can be studied by its autocorrelation function [8]. In this letter, the autocorrelation function of the probe channel is derived analytically to find the root-mean-squared (rms) bandwidth of the spectral broadening. Even for fiber with dispersion compensation, when the power spectra are broadened in WDM systems, up to a certain limit, the power spectra of two channels are overlapped, resulting in interchannel interference. The rms bandwidth is important to study the limitation on number of channels and launched optical power. The calculated values are found to match quite well with the experimental measurements based on a heterodyne method.

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II. Derivation of rms Bandwidth from XPM

Assuming a randomly modulated pump channel and a CW-probe channel, a formal approach to determine the spectral broadening is to solve the following equation for the probe signal [4]:

$$\frac{\partial A_p}{\partial z} + \frac{1}{v_p}\frac{\partial A_p}{\partial t} + \frac{\alpha}{2}A_p = i\gamma(|A_p|^2 + 2|A_{\rm sig}|^2)A_p \qquad (1)$$

where A_p and $A_{\rm sig}$ are the electric field amplitude for probe and pump channel with proper normalization such that optical power is equal to $|A_p|^2$ or $|A_{\rm sig}|^2$, respectively, v_p is the group velocity of the probe signal, α is the fiber loss coefficient, and $\gamma = 2\pi n_2/\lambda_p A_{\rm eff}$ is the nonlinear coefficient where n_2 is nonlinear refractive index coefficient, λ_p is the wavelength of probe signal, and $A_{\rm eff}$ is the effective core area of the optical fiber. Fiber dispersion does not affect the spectral density and the term with dispersion coefficient of β_2 is ignored in (1). As the probe is usually a small signal, the term $|A_p|^2$ in (1) can be neglected and the solution of $A_p(z,t)$ can be obtained adopting a similar approach used in [4]

$$A_p(z,t) = A_p e^{-\frac{\alpha z}{2}} \times \exp\left[-2i\gamma\int_0^z \left|A_{\rm sig}\left(t - \frac{z}{v_p} - d_{\rm sp}z', 0\right)\right|^2 e^{-\alpha z'}\, dz'\right] \qquad (2)$$

where $d_{\rm sp} = 1/v_s - 1/v_p$ is the group velocity mismatch between the pump and probe channels, and $d_{\rm sp} \approx D\Delta\lambda$, where D is the dispersion coefficient and $\Delta\lambda$ is the wavelength separation. Similar to the approach of [4], [7], the solution of (2) implies that fiber dispersion induces only pulse walkoff but no pulse distortion because the pulse spectrum is usually much smaller than the channel separation. For a system modulated by random binary data, the pump signal is

$$P_{\rm sig}(t)|_{z=0} = |A_{\rm sig}(t,0)|^2 = \sum_{k=-\infty}^{+\infty} b_k p(t-kT)$$

where $b_k = \{0, 1\}$ is random binary data, $p(t)$ is the pulse shape, and T is the bit interval. Neglecting the constant factor of $A_p e^{-\alpha z/2}$, the rms bandwidth can be derived from the phase of (2):

$$v(t,z) = e^{-i\varphi(t,z)} \tag{3}$$

where

$$\varphi(t,z) = \sum_{k=-\infty}^{+\infty} b_k q\left(t - \frac{z}{v_p} - kT\right) \tag{4}$$

and

$$q(t) = 2\gamma \int_0^z p(t - d_{\rm sp} z') e^{-\alpha z'}\, dz'. \tag{5}$$

The signal of $v(t,z)$ in (3) is a cyclostationary random process and its average autocorrelation function is

$$\begin{aligned}\bar{R}(\tau) &= \frac{1}{T}\int_0^T E\left\{e^{-i[\varphi(t+\tau)-\varphi(t)]}\right\} dt \\ &= \frac{1}{T}\int_0^T E\left\{\exp\left[-i\sum_{k=-\infty}^{+\infty} b_k[q(t+\tau-kT)\right.\right. \\ &\qquad \left.\left. - \, q(t-kT)]\right]\right\} dt. \end{aligned} \tag{6}$$

Because all binary data b_k are independent of each other, the above average autocorrelation function in (6) becomes

$$\bar{R}(\tau) = \frac{1}{T}\int_0^T \prod_{k=-\infty}^{+\infty} \frac{1}{2}\left\{1 + e^{-i[q(t+\tau-kT)-q(t-kT)]}\right\} dt. \tag{7}$$

The power spectral density is the Fourier transform of the average autocorrelation function and the rms bandwidth is

$$W_{\rm rms}^2 = \frac{\int_{-\infty}^{\infty} \Omega^2 S(\Omega)\, d\Omega}{\int_{-\infty}^{\infty} S(\Omega)\, d\Omega} = -\frac{d^2}{d\tau^2}\bar{R}(\tau)\bigg|_{\tau=0}. \tag{8}$$

After some algebra, the rms bandwidth (8) becomes

$$W_{\rm rms}^2 = \frac{1}{4T}\int_{-\infty}^{\infty}\left[q'(t)q'(t) + q'(t)\sum_{k=-\infty}^{+\infty} q'(t-kT)\right] dt \tag{9}$$

where $q'(t) = dq(t)/dt$. Using some properties of Fourier transform, from (9), we have

$$W_{\rm rms}^2 = \frac{1}{8\pi T}\left\{\int_{-\infty}^{+\infty}\Omega^2 |Q(\Omega)|^2\, d\Omega + \frac{1}{T}\sum_{k=1}^{\infty}\left(\frac{2\pi k}{T}\right)^2 \left|Q\left(\frac{2\pi k}{T}\right)\right|^2\right\} \tag{10}$$

and the relation between $Q(\Omega)$ and $P(\Omega)$ follows directly from (5):

$$|Q(\Omega)|^2 = \frac{4\gamma^2 |P(\Omega)|^2\left[(1 - e^{-\alpha z})^2 + 4e^{-\alpha z}\sin^2\left(\frac{d_{\rm sp}\Omega z}{2}\right)\right]}{\alpha^2 + (d_{\rm sp}\Omega)^2} \tag{11}$$

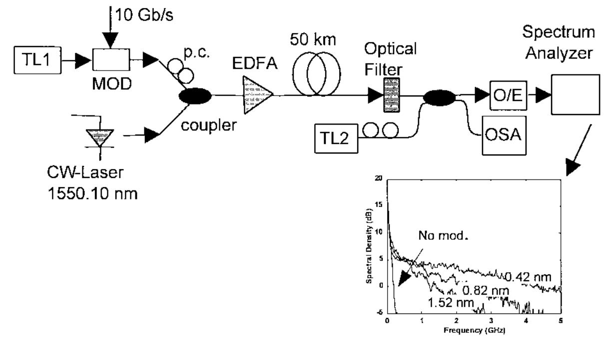

Fig. 1. Experimental setup to measure rms bandwidth using heterodyne method.

where $Q(\Omega)$ and $P(\Omega)$ are the Fourier transform of $q(t)$ and $p(t)$, respectively. For dispersive fiber with large walkoff parameter $d_{\rm sp}$, $Q(\Omega)$ is a low-pass response having a 3-dB bandwidth of approximately $\alpha/d_{\rm sp}$ and the second term of (10) is small compared to the first term. Due to the low-pass characteristic, the rms-bandwidth decreases with the increase of the walkoff parameter $d_{\rm sp}$.

Obviously, the spectrum of $P(\Omega)$ is linearly proportional to the launched optical power. Both $Q(\Omega)$ and $W_{\rm rms}$ are also linearly proportional to the launched optical power. However, this linear dependence of launched optical power is not the same as the linear relationship of [4]. The linear relationship of [4] indicates that $\varphi(t,z)$ in (3) is linearly depending on the input signal. However, the rms-bandwidth is for $v(t,z)$ which is phase-modulated by $\varphi(t,z)$ and neither linearly depending on $\varphi(t,z)$ nor the input signal. Using the famous Carson's rule [9] for frequency modulation, to a certain extend, the bandwidth is proportional to the modulation index which is proportional to the launched optical power for XPM. Although a simple expression is not available, the rms bandwidth can be calculated using numerical integration using the expression of (10) with $|Q(\Omega)|^2$ given by (11).

III. Experimental Verification

Fig. 1 shows the experimental setup to measure the rms bandwidth of XPM induced spectral broadening. A coupler is first used to combine the CW-probe laser emitting at a fixed wavelength of 1550.10 nm and the pump channel (tunable laser TL1) externally modulated by 10-Gb/s $2^{23}-1$ pseudorandom binary sequence (PRBS). The wavelength of the tunable laser is dithered with 10-kHz sinusoidal tone to avoid stimulated Brillouin scattering (SBS). The total optical power at the coupler output is about 0 dBm, with the probe channel about 20 dB weaker than that of the pump. Both pump and probe have the same polarization to maximize the effect of XPM. The combined signal and probe channels are amplified through an erbium-doped fiber amplifier (EDFA) having maximum output power of 17.6 dBm. Standard single-mode fiber of 50 km is used for XPM testing. Heterodyne method is used to measure the power spectrum of the probe signal by beating with a tunable laser (TL2). The intermediate frequency (IF) is about 14.2 GHz with TL2 having shorter wavelength. The wavelength separation is measured with an optical spectrum analyzer (OSA).

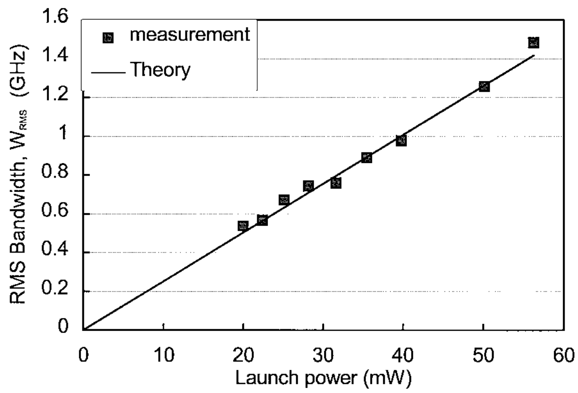

Fig. 2. The rms bandwidth as a function of launch power for a wavelength separation of 0.8 nm.

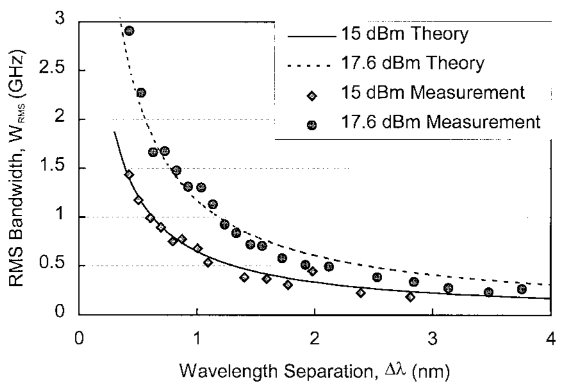

Fig. 3. The rms bandwidth as a function of wavelength separation for launch power of 15 and 17.6 dBm.

The inset of Fig. 1 also shows some measured spectral density with relative unit in dB. The launch power is 17.6 dBm with wavelength separation of 0.42, 0.82, and 1.54 nm. Fig. 1 shows one-side of the spectral density and the frequency is shifted such that the IF corresponding to the zero frequency. The spectral density without modulation is also shown for comparison. From the inset of Fig. 1, the spectral density broadens with the decrease of wavelength separation which effectively decreases the walkoff parameter d_{sp}.

The power spectrum of XPM induced spectral broadening is $a\delta(\Omega) + bG(\Omega)$ where a, b are constants, $\delta(\Omega)$ is delta function and $G(\Omega)$ can be assumed Gaussian shape [10]. Because of the unavoidable linewidth due to the combined phase noise of the CW-laser and tunable laser (TL2), the measured spectra in Fig. 1 are $aL(\Omega)+bG(\Omega)*L(\Omega)$, where $L(\Omega)$ is the Lorentzian line-shape due to laser phase noise, and $*$ denotes convolution. The line-shape of $L(\Omega)$ can be measured with unmodulated pump channel as shown in Fig. 1, resulting with a linewidth of about 24 MHz. The optimized parameters, a, b, σ can be estimated to fit the measured spectral density, where σ^2 is the variance of $G(\Omega)$. The rms bandwidth can then be evaluated according to $\sigma/(1 + a/b)^{1/2}$.

Figs. 2 and 3 show the measured and calculated rms bandwidth. The theoretical results are evaluated using a fiber nonlinear coefficient $\gamma = 2.27$ /km/W (corresponding to $n_2 = 2.8\times10^{-20}$ m^2/W and $A_{\mathrm{eff}} = 50\,\mu\mathrm{m}^2$), a fiber dispersion D of 16.5 ps/km/nm, and a fiber loss coefficient α of 0.2 dB/km. The theoretical calculation assumes an input pulse $p(t)$ of nonreturn-to-zero (NRZ) rectangular shape. Fig. 2 shows that the rms bandwidth is proportional to the launch power to the fiber, or the output power of EDFA. Both calculation and experiment uses a wavelength separation of 0.8 nm, corresponding to frequency separation of 100 GHz. Fig. 3 shows that the rms bandwidth decreases with the increase of wavelength separation for two pump power levels of 15 and 17.6 dBm.

Figs. 2 and 3 show small discrepancy between experimental and theoretical results. The discrepancy could be attributed to the inaccuracy of α, γ, D, the neglecting of fiber dispersion inducing pulse distortion for pump channel, the use of rectangular pulse shape instead of a measured pulse shape, zero-chirp input pulse, etc. Overall, the experimental results match well with theoretical calculation.

IV. Conclusion

An analytical formula is derived for the rms bandwidth of XPM induced spectral broadening from a randomly modulated pump channel to a CW probe channel. The average autocorrelation function of the probe channel is first derived and the rms bandwidth is calculated from the autocorrelation function. The rms bandwidth formula is found to match well with experimental measurements using heterodyne method.

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