Modeling of Waveform Distortion Due to Optical Filtering

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***Abstract*—The usable bandwidth of an optical filter is not only limited by signal attenuation, but also the waveform distortion when optical signal is passed through the edge of the filter. The waveform distortion due to optical filtering is investigated with the assumption of linearly chirped Gaussian pulse. While the first-order optical filtering (linear slope in dB) does not induce waveform distortion, second-order optical filtering induces pulse distortion similar to fiber dispersion.**

***Index Terms*—Filter distortion, optical filtering, wavelength-division multiplexing (WDM).**

I. Introduction

OPTICAL networks using wavelength-division-multiplexed (WDM) technologies can fully utilize the enormous bandwidth of a single-mode optical fiber. In WDM systems, optical filter is an essential device to select the channel, reject noise, attenuate channel power, etc. Optical filtering can be found in WDM multiplexer/demultiplexer, wavelength router, optical cross-connect, and other WDM components. Optical filter is made by various technologies, including array-waveguide grating [1], fiber Bragg grating [2], multilayer interference filter [3], planar grating, Fabry–Perot filter, acousticoptical filter, and others.

Usually, the WDM channel is located at the center of the optical filter and the filter response is flat in that center region. Considerable efforts are taken to design optical filter with two contradictory criteria: a wide flat center region, but narrow bandwidth [4]. To have a large crosstalk rejection ratio, the usable bandwidth of the optical filter is usually limited and the WDM channel may locate at a nonflat region of an optical filter due to wavelength misalignment. To understand the usable bandwidth of an optical filter, studies on distortion induced by filters are required. While most previous works [4]–[7], other than [8] and [9], focused on phase distortion, this paper studies amplitude filtering induced waveform distortion (see Fig. 1). Analytical formulas are derived for the special case of linearly chirped Gaussian input pulse. The optical filter transfer function (in dB scale) is modeled by Taylor series to the second order. Like linear phase shift versus frequency [4] induces only time delay, linear filter slope in dB scale induces optical pulse gain/loss without waveform distortion. Similar to fiber dispersion, the second-order filter distortion induces pulse narrowing or broadening.

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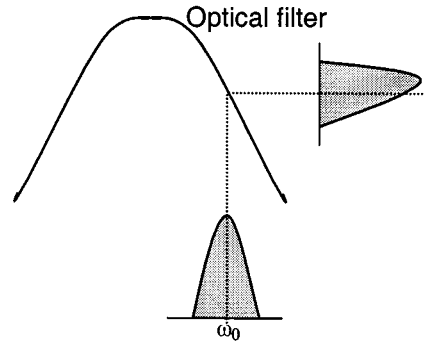

Fig. 1. Distortion induced by optical filtering.

The remaining part of this paper will provide analytical derivation and numerical results for waveform distortion due to optical filtering.

II. Analysis of Optical Filtering-Induced Pulse Distortion

In the analysis of pulse distortion due to fiber (or filter) dispersion, the propagation constant $\beta(\omega)$ is usually expanded by Taylor series to the second order, corresponding to group velocity and dispersion coefficient. The analysis here assumes that both the amplitude response (in dB) and phase response of the optical filter can be expanded using Taylor series to the second order. A close-form analytical result can be derived for linearly chirped Gaussian input pulse.

In Fig. 1, an optical signal is passed through an optical bandpass filter at the "edge." The amplitude response (in dB) of the optical filter can be expressed using Taylor series of $A_0 + A_1(\omega-\omega_0) + A_2(\omega-\omega_0)^2/2 + \cdots$, where ω_0 is the center frequency of the input optical signal, which is usually not equal to the center frequency of the optical fiber A_k, and $k \geq 0$ are the Taylor series coefficients in dB of optical power. Without loss of generality, assume that $A_0 = 0$ dB. The response of the optical filter in linear unit of electric field is

$$H(\omega) = \exp\left[\alpha_1(\omega-\omega_0) + \frac{\alpha_2 + j\beta_2}{2}(\omega-\omega_0)^2 + \cdots\right] \quad (1)$$

where $\alpha_k = A_k \log(10)/20 = 0.115A_k$ and β_2 is the dispersion of the optical fiber. The linear phase, usually expressed by the coefficient of β_1, is ignored because it just induces a constant time delay.

Assuming that the electric field entering into the optical filter is $E_{\text{in}}(t) = A_i(t)e^{-j\omega_0 t}$ and the output electric field is

$E_{\rm out}(t) = A_o(t)e^{-j\omega_0 t}$, the spectrum of the output electric amplitude is

$$A_o(\omega) = A_i(\omega)\, \exp\left[\alpha_1\omega + \frac{\alpha_2 + j\beta_2}{2}\,\omega^2 + \cdots\right] \quad (2)$$

where $A_i(\omega)$ and $A_o(\omega)$ are the Fourier transform of $A_i(t)$ and $A_o(t)$, respectively. For various input waveform, $A_i(\omega)$ can be evaluated by either numerical or analytical form. The output $A_o(\omega)$ can be found by passing $A_i(\omega)$ though the transfer function of $H(\omega)$. The output pulse shape can be calculated by taking the inverse Fourier transform of $A_o(\omega)$. For most pulse shapes, neither $A_i(\omega)$ nor the inverse of $A_o(\omega)$ can be evaluated analytically.

Lengthily, numerical simulation [8], [9] is necessary for an accurate estimation of the system penalty generated from optical filtering. Numerical simulation usually provides accurate results, but without insight of the problem. As a simple example with viable analytical results, the input optical pulse can be assumed a normalized linearly chirped Gaussian pulse of

$$A_i(t) = \exp\left[-\frac{1+jC}{2}\left(\frac{t}{T_0}\right)^2\right] \quad (3)$$

where C and T_0 are the chirp parameter and the $1/e$ pulsewidth, respectively, of the input pulse. Linearly chirped pulse is assumed here to model the output pulses from external modulators with nonzero chirp parameter [10], [11]. Considering up to the second-order Taylor series coefficients, after some algebra, the output spectrum is

$$A_o(\omega) = \sqrt{\frac{2\pi T_0^2}{1+jC}}\, \exp\left[\alpha_1\omega + \left(\alpha_2 + j\beta_2 - \frac{T_0^2}{1+jC}\right)\frac{\omega^2}{2}\right]. \quad (4)$$

Taking the inverse Fourier transform, the output electric field is

$$A_o(t) = \frac{T_0}{[T_0^2 - (\alpha_2 + j\beta_2)(1+jC)]^{1/2}} \cdot \exp\left\{-\frac{(1+jC)(t+j\alpha_1)^2}{2\,[T_0^2 - (\alpha_2+j\beta_2)(1+jC)]}\right\}. \quad (5)$$

The output pulse shape in optical power is $p_o(t) = |A_o(t)|^2$. After some algebra

$$p_o(t) = \frac{T_0^2}{D^{1/2}}\, \exp\left\{-\frac{\Gamma_1(t^2-\alpha_1^2) - 2\alpha_1\Gamma_2 t}{D}\right\} \quad (6)$$

where

$$D = \left(T_0^2 + \beta_2 C - \alpha_2\right)^2 + (\beta_2 + \alpha_2 C)^2 \quad (7a)$$

$$\Gamma_1 = T_0^2 - \alpha_2\left(1 + C^2\right) \quad (7b)$$

$$\Gamma_2 = \beta_2\left(1 + C^2\right) + CT_0^2. \quad (7c)$$

From (6), the output pulse is still a Gaussian pulse. The $1/e$-width of the input pulse is T_0 and that of the output Gaussian pulse becomes

$$T_{\rm out} = \sqrt{\frac{D}{\Gamma_1}}. \quad (8)$$

From (5) of output optical pulse, the term of α_1 induces an "imaginary" time shift, corresponding to the "real" time shift provided by β_1. While there is nothing corresponding to "imaginary" time shift in physics, for most pulses, α_1 just gives a constant gain or loss of the optical pulse. For example, if $\alpha_2 = \beta_2 = C = 0$, nonzero α_1 provides a uniform gain of $\exp(\alpha_1^2/T_0^2)$.

For $\alpha_2 = 0$, the formulas of (6)–(8) are identical to those of [12] and [13] for pulse broadening due to fiber dispersion. With $\beta_2 = C = 0$, the output pulse may be narrowed or broadened, according to

$$\frac{T_{\rm out}}{T_0} = \left[1 - \frac{\alpha_2}{T_0^2}\right]^{1/2}. \quad (9)$$

For zero-chirp pulse ($C = 0$), an optical filter could provide either pulse narrowing or broadening, depending on the sign of α_2 or the curvature of the filter. When α_2 approaches T_0^2, high-order terms of α_k, $k > 2$, contribute to the signal and limit the pulsewidth. However, this paper will not consider those high-order terms in details. For linearly chirped pulse with $\beta_2 = 0$, the $1/e$-width of output pulse is

$$\frac{T_{\rm out}}{T_0} = \left[1 - \frac{\alpha_2}{T_0^2} + \frac{\alpha_2 C^2}{T_0^2 - \alpha_2(1+C^2)}\right]^{1/2}. \quad (10)$$

For positive α_2, the pulsewidth is always larger than that without chirp. It is still possible to archive pulse compression for $|C| < 1$, with the shortest pulse width of

$$\left.\frac{T_{\rm out}}{T_0}\right|_{\rm min} = \left[\frac{2|C|}{1+C^2}\right]^{1/2} \quad (11)$$

when $\alpha_2 = (1-|C|)T_0^2/(1+C^2)$. For $|C| > 1$, the output pulse is always broadened.

For α_1 equal to zero, the peak power of the pulse is $p_o(0)$. With nonzero α_1, the peak power of the pulse is $p_o(t_m)$, where $t_m = \alpha_1\Gamma_2/\Gamma_1$ is the center of the output Gaussian pulse. It is important to find the power penalty attributed to filter generated distortion. The eye-diagram penalty is approximately equal to

$$\delta P \approx \left[1 + 2\,\frac{p_0(t_m+T)}{p_0(t_m)}\right]\frac{T_{\rm out}}{T_0} \quad (12)$$

where T is the bit-interval and $1/T$ is the data rate, $p_o(t_m + T)/p_o(t_m)$ is the intersymbol interference to signal ratio, and $T_{\rm out}/T_0$ is also the ratio of pulse power. In the above expression, we neglect the intersymbol interference from $p_o(t_m+2T)$, $p_o(t_m + 3T)$, etc. After some algebra

$$\delta P \approx \left[1 + 2\,\exp\left(-\frac{T^2}{T_{\rm out}^2}\right)\right]\frac{T_{\rm out}}{T_0} \quad (13)$$

which is independent of α_1. For systems without signal dependent noise, the power penalty is also equal to the eye-diagram penalty. Usually, the power penalty is expressed in dB scale, given by $10\,\log_{10}(\delta P)$.

The power penalty defined by (13) neglects the effect of receiver filter and the noise bandwidth of the receiver. If a Gaussian shape receiver filter with $1/e$-bandwidth of $1/T_f$ is used, the parameter of $T_{\rm out}$ in (13) should be replaced by

$[T_{\rm out}^2 + T_f^2]^{1/2}$. In the later part of this paper, the effects of receiver filter are ignored.

III. Numerical Results

Fig. 2 shows the power penalty of (13) and $T_{\rm out}/T_0$ of (10) as a function of A_2/T^2, where A_2 is proportional to α_2 in (1) and calculated by $A_2 = d^2H(\omega)/d\omega^2|_{\omega=\omega_0}$, with $H(\omega)$ in dB scale of optical power. Usually, except fiber Bragg grating based filters, β_2 of an optical filter is very small [4] and $\beta_2 = 0$ is assumed in Fig. 2. Having $\beta_2 = 0$, both the output pulsewidth and the power penalty depends only on C^2, which is independent of the sign of the chirp parameter. In Fig. 2, the $1/e$-width of the input pulse is $T_0 = 0.45T$, corresponding to a full-width half-maximum (FWHM) pulsewidth of $0.75T$. The chirp parameter of the input pulse is $C = 0$ and ± 0.75. The value of $C = \pm 0.75$ is approximately equal to the chirp parameter of an external modulator [11].

From Fig. 2, both the pulsewidth and the power penalty increase when A_2/T^2 becomes more negative. A negative A_2/T^2 is usually for the case of a bandpass filter. As the curvature of the bandpass filter increases, it induces larger waveform distortion to the optical signal, broadens the pulse, provides higher intersymbol interference, and thus a higher power penalty. As implied by (9), both pulsewidth and the power penalty decreases for a positive A_2/T^2 for a zero-chirp optical pulse. A positive A_2/T^2, usually for the case of a band-rejection filter, may compress the optical pulse and decrease the power penalty for zero-chirp pulse. However, with a chirp parameter of $C = \pm 0.75$, a positive A_2/T^2 broadens the optical pulse and induces very high power penalty for $A_2/T^2 > 1$ dB. An optimal positive A_2/T^2 exists that achieves the shortest pulsewidth of (11) and provides pulse narrowing. The power penalty decreases with the increase of the chirp parameter for bandpass filter having negative A_2/T^2. The power penalty increases with the increase for chirp parameter for band-rejection filter having positive A_2/T^2.

Fiber–Bragg grating has dispersion around the center region [4]. Fig. 3 shows the power penalty and pulsewidth as a function of A_2/T^2 for $\beta_2/T^2 = 0.1$. In general, the power penalty increases with the increase of chirp parameter from negative to positive, especially in the positive A_2/T^2 region. In the case of bandpass filter having negative A_2/T^2, as both parameters of D and Γ_1 in (7) depend on α_2, β_2, and C with a complex relationship, the power penalty increases as the chirp parameter changing from negative to positive. However, after the chirp parameter larger than a certain value, the power penalty decreases again. The power penalty for the case of $\beta_2/T^2 = -0.1$ is the same as that in Fig. 3 with the curves of positive and negative chirp exchanged.

A WDM system may pass through many fiber spans and optical filters. The overall fiber and filter dispersion can be equalized using dispersion compensation units, i.e., to make the overall β_2 approaching zero. In the operating region of the optical filter, the overall distortion coefficient must be in the region of $-2.5 < NA_2/T^2 < 1$ for less than 3-dB power penalty as from Fig. 2, where N is the total number of optical filters and the positive side is not the case for bandpass filter. For a 10-Gb/s system, the requirement becomes

$$N|A_2| < 0.025 \text{ dB/(GHz)}^2. \quad (14)$$

In other words, the filter cannot change from flat region with 0 dB/GHz to nonflat region with 0.025 dB/GHz within a frequency separation of 1 GHz, and so on. Given the maximum number of optical filters that the system may pass through, the operating region of the optical filter must conform to the criterion of (14) for a small filter curvature. From Fig. 3, the requirement of (14) will be tightened if some portions of fiber or filter dispersion are unequalized.

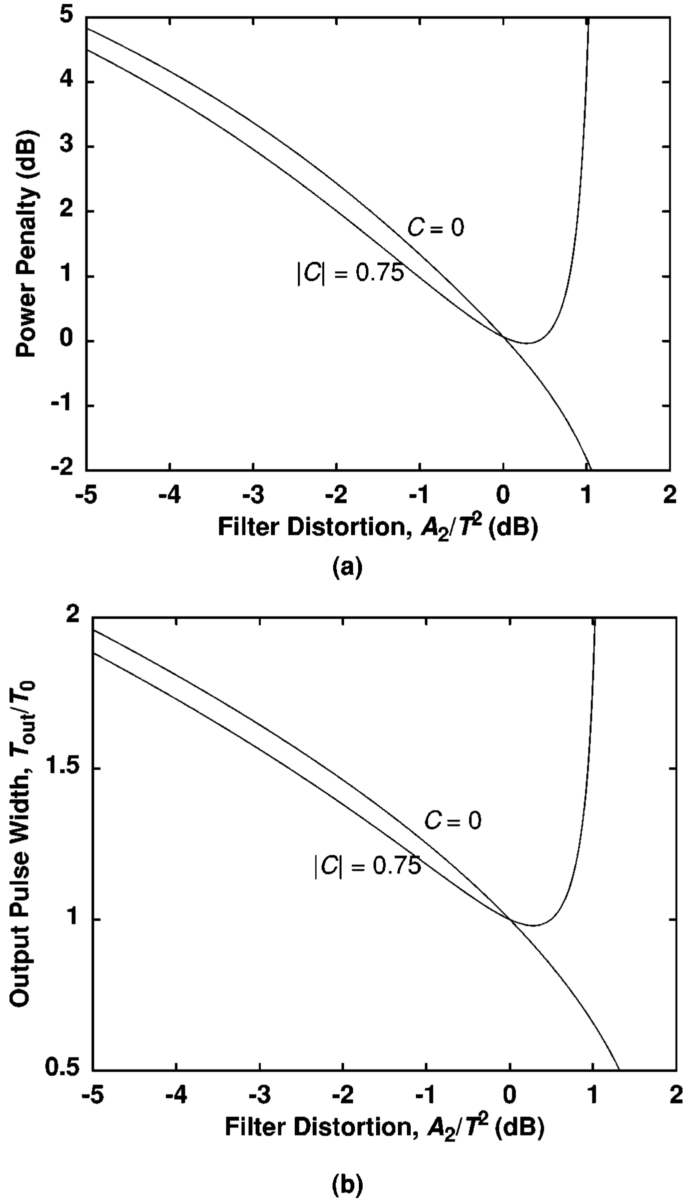

Fig. 2. (a) Power penalty and (b) pulsewidth as a function of normalized filter-distortion coefficient A_2/T^2 for nondispersive filter with $\beta_2 = 0$.

IV. Conclusion

This paper provides a model of waveform distortion due to optical filtering. Linearly chirped Gaussian pulse is assumed

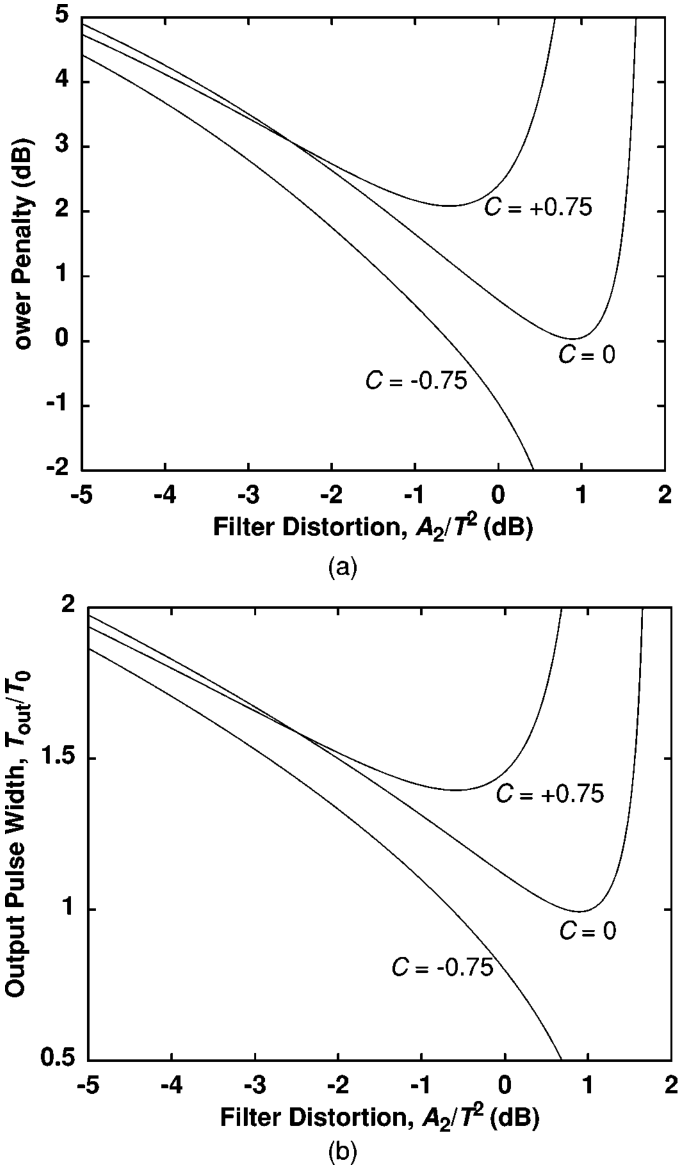

Fig. 3. (a) Power penalty and (b) pulsewidth as a function of normalized filter-distortion coefficient A_2/T^2 for dispersive filter having $\beta_2/T^2 = 0.1$.

to provide some insight of the problem. It is found that the first-order filter distortion, linear slope in dB scale, provides no distortion to Gaussian pulse. The output pulse is still a Gaussian pulse with the second-order distortion. Optical filtering can provide both pulse narrowing and broadening, depending on the values of filter parameters, pulsewidth, and chirp parameter of input pulse. Although a positive second-order filter coefficient of A_2 is usually providing narrower pulse shape or smaller power penalty, a bandpass filter normally has a negative filter coefficient of A_2. In addition to designing a filter with linear phase, this paper suggests a new design criterion (14) for the second-order filter coefficient in the operating region of the optical filter.

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