Performance Analysis of Multi-Dimensional Optical Routing Networks

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Abstract

With the recent explosive growth of Internet traffic, optical networks with high connectivity and large capacity are indispensable. This can be achieved by multi-dimensional optical routing networks, which can offer fine bandwidth granularity and a large number of channels. In this paper, a trunk switched model is used to study the performance of multi-dimensional optical routing networks. The closed-form solutions are derived for the utilization gain by multiplexing (adding new dimensions in the network). The network utilization gain is found to be closely related to the link correlation and be insensitive to the network size. Besides, the closed-form conversion gains are derived for both partially and fully convertible cases. The utilization gain by multiplexing is compared with the conversion gain for the wavelength routing network. The results provide crucial information in performance optimization of multi-dimensional optical routing networks.

Index Terms — multi-dimensional optical routing network, trunk switching, link correlation, conversion gain.

1 Introduction

With the rapid growth of Internet users and service classes, future optical networks are desirable to offer high connectivity and large capacity. Wavelength routing networks, which employ wavelength division multiplexing (WDM), are the most promising solution for the near-term implementation of high capacity IP network infrastructure. Further improvement in performance, such as network utilization and blocking, can be achieved by several approaches. Firstly, wavelength conversion can be used to improve the flexibility of constructing an end-to-end lightpath [1, 2, 3]. Secondly, performance can be improved by adopting routing and wavelength assignment (RWA) schemes [4, 5, 6] at the expense of sophisticated control and management. Besides, by providing finer bandwidth granularity, more channels can be provided. Recent advances in Dense Wavelength Division Multiplexing(DWDM) enable the provision of more bandwidth as well as a larger number of channels. However, with the ever increase in the network size and the number of users, the limited number of available wavelengths will be exhausted eventually. As an example, consider a 100-node network employing shortest-path-first-fit (SPFF) RWA scheme. If each node has 16 destination nodes, the number of wavelengths required is more than 200 [7]. By extending the channels to other dimensions such as time domain, frequency domain and code domain, the total number of available channels can be increased substantially. Thus, instead of using mere WDM technique, hybrid combination of WDM with optical time-division multiplexing (OTDM), optical code-division multiplexing (OCDM) and/or sub-carrier multiplexing (SCM), can offer large number of channels and fine channel granularity.

In [7, 8, 9], the blocking performance of two-dimensional optical networks was studied. In this paper, the performance of a generalized multi-dimensional optical routing network is studied by a trunk switched model, including the link utilization gain and the optical conversion gain. The paper is organized as follows. In Section

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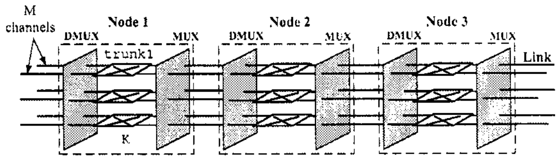

Figure 1: Path in a homogeneous trunk switched network with $K = 3, M = 2$, K: trunk number, M: trunk size.

2, trunk switched networks are introduced. In Section 3, the analytical model is proposed for studying the performance of a multi-dimensional optical routing network with homogeneous switches. Based on this model, in Section 4, we present the network utilization gain by comparing the link utilization of a multi-dimensional network to that of a one-dimensional network. In Section 5, the conversion gain by employing optical conversion is derived. In Section 6, the utilization gain from adding new dimensions is compared with the conversion gain for a wavelength routing network. Section 7 concludes the paper.

2 Trunk switched network

To study the performance of OTDM/WDM networks, trunk switched model was first introduced in [8]. In a trunk switched network, channels on each link are grouped into several trunks according to the conversion capability of a node. Each trunk employs a full-channel interchanger (FI) to interchange all the channels within the same trunk. ***Homogeneous trunk switched network*** is a class of the trunk switched network in which each node has identical conversion capability. An example path is illustrated in Fig. 1, where there are K trunks on each link and each of which has M channels.

Here, we generalize the homogeneous trunk switched networks to model the n-dimensional ($n \geq 1$) optical routing networks. For an n-dimensional network, the number of channels on the i^{th} dimension is N_i ($i = 1, ..., n$) and the number of convertible dimensions in the network is given by m, where $0 \leq m \leq n$. That is, m out of the n dimensions are convertible, thus the channels on these m dimensions can be fully interchanged and considered as a *trunk*. Here, partial conversion on one dimension is not considered. Therefore, the network is modeled as a homogeneous trunk switched network with

$$M = \prod_{i \in E} N_i, \; K = \prod_{i \in \bar{E}} N_i, \tag{1}$$

where E is the set of m convertible dimensions, and $\bar{E}$ is the set of $(n - m)$ non-convertible dimensions.

For instance, an OTDM/WDM network with L fibers per link, F wavelengths per fiber and T time slots per wavelength can be modeled as a homogeneous trunk switched network with $M = LT$ and $K = F$ if optical time slot interchangers and fiber switches are employed.

3 Analytical Model

Based on the concept of trunk switching and the model in [3, 10] for studying wavelength routing networks, we propose an analytical model to evaluate the performance of the homogeneous multi-dimensional optical routing networks. Consider a circuit switched mesh network where the channel occupancy status are independent. We assume all channels have the same busy probability ρ, and, the number of busy channels on a link is binomial distributed. When all the channels in a trunk are occupied, the trunk is regarded as *busy*. Otherwise, the trunk is *idle*.

In optical routing networks, the traffic originates from the source node may traverse more than one hop over the network and this leads to *link correlation* among the successive links. Basically, link correlation ϕ represents

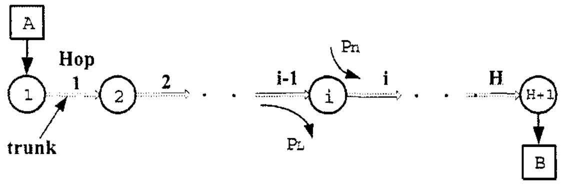

Figure 2: An H-hop call request.

the probability that a call is forwarded to the next hop along a path. Therefore, $(1-\phi)$ gives the probability that a call is dropped at one node.

As shown in Fig. 2, station A requests a session to station B over a mesh network. There are H hops (links) from the source node 1 to the destination node $H+1$. Due to the limited conversion capabilities at the nodes, channel selections on these H hops should be confined to the same trunk for establishing a lightpath. This is regarded as *fixed-trunk constraint.* At node i, given that its input trunk is busy, P_L is the conditional probability that at least one call is dropped from its input trunk, i.e. the trunk becomes idle at node i. For a network with M channels per trunk, P_L can be derived as

$$P_L = \sum_{i=1}^{M} \binom{M}{i} (1-\phi)^i (\phi)^{M-i} = 1 - \phi^M. \tag{2}$$

Similarly, given that the input trunk of node i is idle, P_n is the conditional probability that sufficient new calls are added at node i so as to make its output trunk (on the i^{th} hop) become busy. Extending the previous work in [3, 10] to homogeneous trunk switched networks, the busy probability q_i of the trunk on the i^{th} hop can be derived as

$$q_i = (1-q_{i-1}) P_n + q_{i-1} (1 - P_L + P_L P_n). \tag{3}$$

In a network with K trunks per link and M channels per trunk, there are K end-to-end fixed-trunks. A lightpath can be established in any one of the fixed-trunks, but not across different trunks. Since all channels are assumed to be independent and have the same busy probability ρ, the link utilization of the network is ρ and the busy probability of a trunk on the i^{th} hop is denoted by q_i and is simply

$$q_i = \rho^M, \quad 1 \le i \le H \tag{4}$$

Further, assuming there is no blocking on both the access link (from station A to node 1) and the exit link (from node $H+1$ to station B), the blocking probability for an H-hop call request (say from station A to B) is given by

$$\begin{aligned} P_b &= \left(1 - \prod_{i=0}^{H-1} P_r \left\{ \begin{matrix} \textit{this trunk on} \\ (i+1)^{th}\ \textit{hop} \\ \textit{is idle} \end{matrix} \middle| \begin{matrix} \textit{a trunk on} \\ i^{th}\ \textit{hop} \\ \textit{is idle} \end{matrix} \right\} \right)^K \\ &= \left(1 - (1-P_n)^H \right)^K . \end{aligned} \tag{5}$$

where the 0^{th} hop ($i=0$) represents the access link.

Combining (2)–(5) together, link utilization ρ can be expressed by

$$\rho(M,K,H) = \left(\frac{1 - \left(1 - P_b^{\frac{1}{K}}\right)^{\frac{1}{H}}}{1 - \left(1 - P_b^{\frac{1}{K}}\right)^{\frac{1}{H}} \Phi(M)} \right)^{\frac{1}{M}}, \tag{6}$$

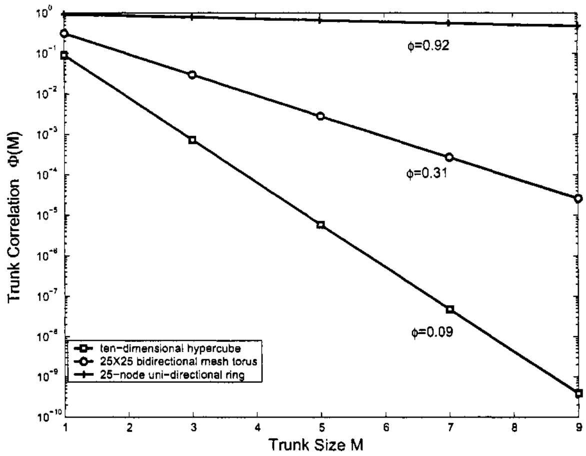

Figure 3: Traffic correlation versus the trunk size in a trunk switched network

where $\Phi(M) \triangleq 1 - P_L = \phi^M$ and is defined as *trunk correlation* of the network. Note that, as trunk size M becomes large, $\Phi(M)$ approaches zero. Therefore, the link independent assumption is valid only for the networks with large values of M. This verifies the approximation applied in the calculation of link utilization for convertible wavelength routing networks in [3].

Fig. 3 shows the relation between trunk correlation $\Phi(M)$ and trunk size M. Three types of networks are considered and their corresponding link correlations [8] are given beside the curves. As shown, trunk correlation $\Phi(M)$ decreases exponentially with respect to trunk size M.

4 Utilization Gain

For an optical network with more than one dimension, it can provide finer bandwidth granularity by dividing the channel bandwidth of a one-dimensional network into fractions. Comparing a generalized n-dimensional optical routing network with a one-dimensional optical routing network with the same topology, the ratio of their link utilizations is defined as *utilization gain* G_u, and is given as

$$G_u \triangleq \frac{\textit{Traffic carried in the n-dimensional network}}{\textit{Traffic carried in the one-dimensional network}}. \qquad (7)$$

For both one- and n-dimensional networks without conversion capabilities, they can be modeled as trunk switched network with unit trunk size (i.e. $M = 1$). For the one-dimensional case, the number of trunk per link, K, equals N_1, where N_1 is the number of channels per link. The link utilization for an H-hop path is denoted by $\rho(1, N_1, H)$, as derived from eq. (6). Assuming $N = \prod_{i=1}^{n} N_i$ is the total number of channels per link for the n-dimensional network, the the number of trunks is $K = N$ and the link utilization of an H-hop path can be denoted by $\rho(1, N, H)$. Assuming the bandwidth on a link is fixed for both networks, the ratio of their link utilizations, or the utilization gain of an n-dimensional network is derived from eq.(6) and eq.(7) as:

$$G_u = \left(\frac{1 - \left(1 - P_b^{\frac{1}{N}}\right)^{\frac{1}{H}}}{1 - \left(1 - P_b^{\frac{1}{N_1}}\right)^{\frac{1}{H}}} \right) \left(\frac{1 - \left(1 - P_b^{\frac{1}{N_1}}\right)^{\frac{1}{H}} \phi}{1 - \left(1 - P_b^{\frac{1}{N}}\right)^{\frac{1}{H}} \phi} \right). \qquad (8)$$

As $N > N_1$ and $\phi < 1$, the utilization gain G_u is greater than 1. Thereby, an optical routing network with finer bandwidth granularities gives a higher link utilization. As the total number of available channels, N,

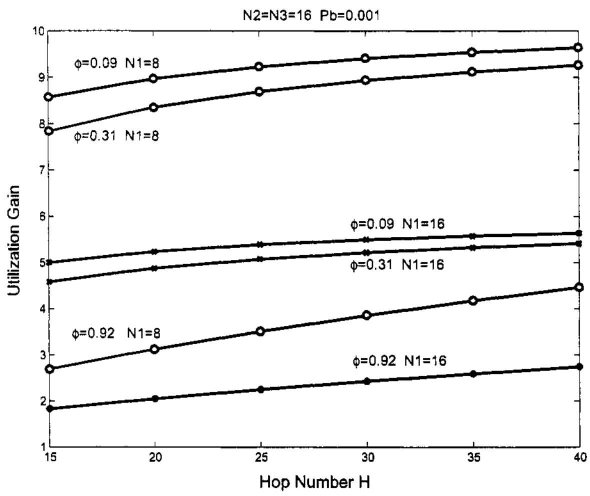

Figure 4: Utilization gain versus hop numbers for the networks with different original channel number N_1

increases, the utilization gain will increase accordingly, but not linearly. When $N \gg 1$, $\rho(1, N, H)$ will be close to 1, and the utilization gain will approach $\frac{1}{\rho(1,N_1,H)}$.

Fig. 4 shows the utilization gain of the n-dimensional network ($n = 3$) versus the number of hops, H. The allowed blocking probability is $P_b = 0.001$ and $N_2 = N_3 = 16$. Three networks with different link correlation ϕ are considered. With the increase in N_1, the utilization gain decreases in all three networks. And the utilization gain reduces with the increase in the link correlation ϕ. It is also shown that the utilization gain is fairly insensitive to the number of hops, H, when $H \geq 20$, disregarding the values of N_1 and ϕ. This result implies the utilization gain of the n-dimensional network is fairly insensitive to the network size if the network is large enough.

5 Conversion Gain

In this section, we apply the homogeneous trunk switched model to investigate the performance improvement of an n-dimensional network when optical conversion is employed. For an n-dimensional network, the number of convertible dimensions before and after the additional conversions are assumed to be m_1 and m_2, respectively, where $0 \leq m_i \leq n$, $i = 1, 2$. For $m_i = 0$, it is a non-convertible network, and for $0 < m_i < n$, it is a partially convertible network, while $m_i = n$ means a fully convertible network. By substituting m_1 into eq.(1), we model the original n-dimensional network as a trunk switched network with K_1 trunks and M_1 channels per trunk. After additional optical conversions are provided, the network can be modeled as a trunk switched network with K_2 trunks and M_2 channels per trunk. Notice that, $M_1K_1 = M_2K_2$ and $M_2 > M_1$. By substituting the parameters $\{M_1, K_1\}$ and $\{M_2, K_2\}$ to eq.(6), we can get the link utilizations before and after the optical conversion, denoted as ρ_1 and ρ_2, respectively. Hence, the closed-form *conversion gain*, which is given by $G_c = \frac{\rho_2}{\rho_1}$, can be derived.

Assuming H is large while K_1 and K_2 have small or moderate values, by making some appropriate simplifications, it can be derived that the maximum value $G_{c,max}$ of the conversion gain is bounded by

$$G_{c,max} < \left(H\left(1 - \phi^{M_1}\right)\right)^{\left(\frac{1}{M_1} - \frac{1}{M_2}\right)}, \quad M_2 > M_1 \tag{9}$$

It is found from eq.(9) that the maximum utilization gain depends on the number of hops H, link correlation ϕ, the former trunk size M_1, and the trunk expanding factor ($S = \frac{M_2}{M_1}$). By keeping all other parameters unchanged, an increase in S will lead to a higher conversion gain. However, with the increase in S, the gain will saturate

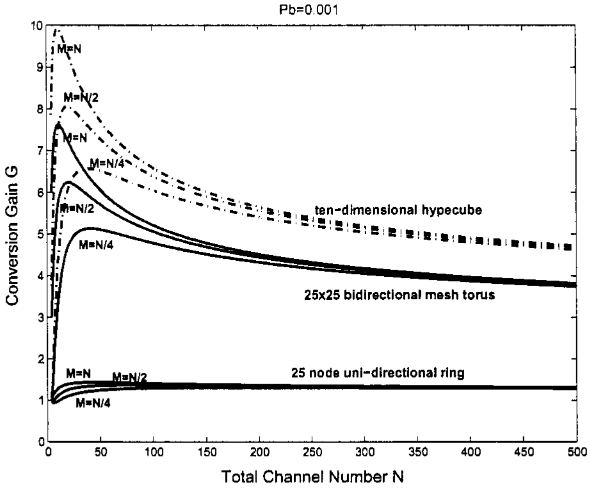

Figure 5: Conversion gain versus total channel number

fast to

$$G_S = \left(H\left(1-\phi^{M_1}\right)\right)^{\frac{1}{M_1}}. \tag{10}$$

Fig. 5 demonstrates the conversion gain when optical converters are added in an n-dimensional network that originally does not have any conversion. Three types of networks with different link correlations are investigated. N is the total channel number. Three cases are considered. $M = N$ is the fully converted case while $M = \frac{N}{2}$ and $M = \frac{N}{4}$ are the partial converted cases. As illustrated in the figure, the network with a higher link correlation exhibits less significant conversion gain under the same scenario. Also, for each type of network, the conversion gain is the highest when fully conversion is provided ($M = N$). However, even when the trunk size is as small as one quarter of the total channel N, the conversion gain is very close to the fully converted case, especially when N is large. This result implies, by providing optical conversion on the dimension even with a small number of channels, the link utilization improvement can be significant.

6 Comparison on the Utilization Gain by Multiplexing and by Conversion

For a wavelength routing network, it can be either upgraded to a convertible wavelength routing network by placing converters at the nodes or upgraded to a 2-dimensional networks by adding a new dimension, TDM, for example. From the derivations in the previous sections, these two approaches can both provide improvements in the link utilization. In this section, we will compare the improvements in the link utilization under these two approaches for a wavelength routing network.

Without the loss of generality, we assuming there are N_1 wavelengths on each link in the wavelength routing network and the number of channels in the additional dimension is N_2. After the addition of the second dimension in the wavelength routing network, by eq.(6) and eq.(7), the utilization gain is given as

$$G_{wu} = \frac{\rho\left(1, N_1 N_2, H\right)}{\rho\left(1, N_1, H\right)}. \tag{11}$$

When by applying wavelength converters into the network, similarly, the conversion gain is derived as

$$G_{wc} = \frac{\rho\left(N_1, 1, H\right)}{\rho\left(1, N_1, H\right)}. \tag{12}$$

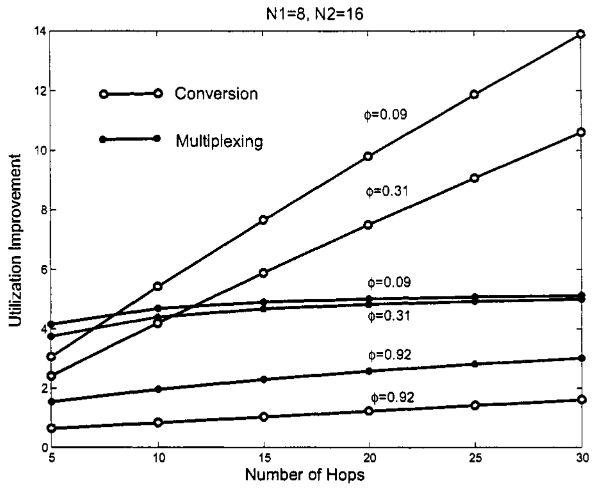

Figure 6: Utilization improvement (by multiplexing or by conversion) versus number of hops H.

Fig.6 shows the improvement in the link utilization of the network versus the number of hops H. The allowed blocking probability is $P_b = 0.001$. N_1 and N_2 are assumed to be 8 and 16, respectively. It is found that, link correlation affects the conversion gain more significantly than the utilization gain by multiplexing (adding a new dimension). For a highly correlated network, as a ring with $\phi = 0.92$, the utilization gain by multiplexing is always larger than the conversion gain, no matter how large the network is. For the moderately ($\phi = 0.31$) and slightly (ϕ=0.09) correlated networks, when the network size is small, the addition of a new dimension can obtain a higher utilization gain than deploying converters.

Fig.7 illustrates the improvement in the link utilization of the network versus the wavelength number N_1, with $N_2 = 16$ and $P_b = 0.001$. For a highly correlated network, no matter how many number of wavelengths (N_1) it has, utilization gain by deploying a new dimension would be higher than the conversion gain. However, for the moderately and slightly correlated networks, only when the number of wavelength is small, the utilization improvement by multiplexing is more significant than the conversion gain.

7 Conclusions

In this paper, an analytic model is proposed for studying the network utilization of generalized multi-dimensional optical routing networks. It is found that the utilization gain by multiplexing is closely related to the link correlation. Less link correlated network will achieve more significant utilization gain. It is also found that, the utilization gain is fairly insensitive to the network size. We derive, for the first time, closed-form solutions for the conversion gain in a partially or fully convertible n-dimensional optical routing network. From the comparison of the utilization gain by multiplexing with the conversion gain, we find that highly correlated networks can obtain higher utilization gain by multiplexing. Also, small networks and the networks with small number of channels also benefit more from multiplexing than from conversion.

Acknowledgment

This work was partially supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region (Project CUHK 4172/01E).

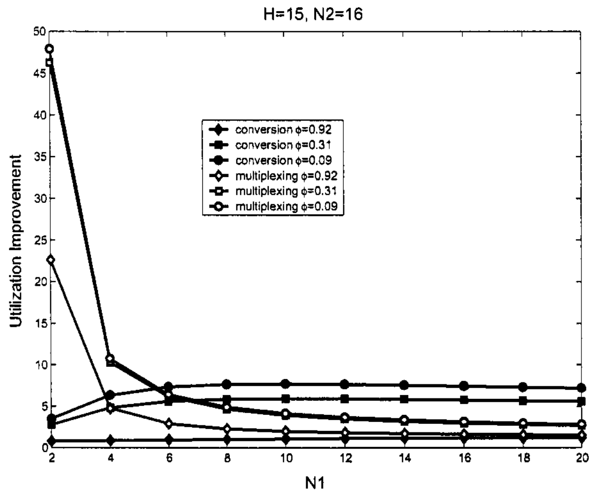

Figure 7: Utilization improvement (by multiplexing or by conversion) versus N1.

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